Ch - 1

Relations And Functions

1. Let $A = \{3, 5\}$. Then, number of reflexive relations on A is

(CBSE 2023)

(a) 2   (b) 4   (c) 0   (d) 8

2. check if the relation R in the set R' of real numbers defined as $R = \{(a, b) : a < b\}$ is

(CBSE 2020)

(a) symmetric   (b) transitive

3. A function $f : A \rightarrow B$ defined as $f(x) = 2x$ is both one-one and onto. if $A = \{1, 2, 3, 4\}$, then find the set B.

(CBSE 2023)

4. if $R = \{(x, y) : x + 2y = 8\}$ is a relation in N, write the range of R.

[AI CBSE 2014]

5. Let $R = [(a, a^3) : a$ is a prime number less than $5]$ be a relation. Find the range of R.

[CBSE (F) 2014]

6. Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2$ divides $(a - b)\}$. write the equivalence class of $\{0\}$.

[CBSE (D) 2014 C]

7. Let $f : \{1,3,4\} \to \{1,2,5\}$ and $g : \{1,2,5\} \to \{1,3\}$ given by $f = \{(1,2),(3,5),(4,1)\}$ and $g = \{(1,3),(2,3),(5,1)\}$. write down $g \circ f$.

[AI CBSE 2014 C]

8. if the functions $f : R \to R$ be given by $f(x) = x^2 + 2$ and $g : R \to R$ be given by $g(x) = \frac{x}{x-1}$, $x \neq 1$, find $f \circ g$ and $g \circ f$ and hence find $(f \circ g)(2)$ and $(g \circ f)(-3)$

[AI CBSE 2014]

9. Let $A = \{1, 2, 3, \dots, 9\}$ and R be a relation in $A \times A$ defined by $(a, b)\ R\ (c, d)$ if $a + d = b + c$ for all $(a, b), (c, d)$ in $A \times A$. Prove that R in an equivalente relation. also, obtain the equivalence class $[(2,5)]$.

[CBSE (D) 2014]

0. show that the function f in $A = R - \{\frac{2}{3}\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. hence find f^{-1}.

[CBSE (D) 2013]

1. Consider $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$ where R_+ is the set of all non-negative real numbers.

[CBSE (F) 2011, AI CBSE 2013]

2. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$. Show that f is one-one and onto. Hence find f^{-1}

[CBSE (D) 2013 C]

13- Let N be the set of natural No. and R be the relation on $N \times N$ defined by $(a,b) R (c,d)$ iff $ad = bc \; \forall a,b,c,d \in N$, show that R is an equivalence relation. (CBSE) 2020

14- If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, find A^{-1} ~~and~~

15- Let $A = \{x \in Z, 0 \le x \le 12\}$, show that $R = \{(a,b) \in A, |a-b|$ is divisible by $4\}$ is an equivalence relation. find the set of all elements related to L. Also write the equivalence class [2] - 2022

16- A function $f : [-4, 4] \to [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$ show that f is an onto function but not a one-one function. Further. find all possible value of 'a' for which $f(a) = \sqrt{7}$ - CBSE-2023

17- Find No of Relations (reflexive) in a set A whose $n(A) = 3$ CBSE-21

18- Show that the relation R on R defined as $R = \{(a,b) : a \le b\}$ is reflexive and transitive but not symmetric CBSE-19

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INVERSE TRIGONOMETRIC FUNCTION

1. The value of $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is (CBSE 2023)

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

2. Find the domain of the following functions: (CBSE 2023)
   (i) $\sin^{-1}(x^2-4)$
   (ii) $\sin^{-1}x + \cos x$

3. Find the principal value of the following: (CBSE 2020, NCERT Exemplar)
   (i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (ii) $\cos^{-1}\left(-\frac{1}{2}\right)$ (iii) $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

4. Evalute each of the following: (CBSE 2023)
   (i) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ (ii) $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$

5. Find the value of (CBSE 2023)

$$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] + \tan^{-1} 1$$

6 The value of $\sin^{-1}\cos(3\pi/5)$ is
(A) $\pi/10$ (B) $3\pi/5$ (C) $-\pi/10$ (D) $-3\pi/5$ (CBSE 2020)

7 – Draw the graph of $\cot^{-1}x$ also writes its range CBSE-2023

8 – Evaluate $\sin^{-1}[\sin(3\pi/4)] + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$ CBSE-2023

MATRICES

(1) If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find matrix A. (CBSE 2019)

(2) If $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$, find the value of $(x-y)$. (CBSE 2019)

(3) Find a matrix A such that $2A - 3B + 5C = 0$, where (CBSE 2019)

$B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

(4) if $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, prove that $(A - 2I)(A - 3I) = O$. (CBSE 2019)

(5) If $A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$, and $I = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, find scalar k so that $A^2 + I = kA$ (CBSE 2020)

(6) Using elementary operations, find the inverse of the following matrices

(i) $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ (CBSE 2019) (ii) $\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$ (CBSE 2019) (iii) $\begin{bmatrix} 2 & 1 & -3 \\ -1 & -1 & 4 \\ 3 & 0 & 4 \end{bmatrix}$ (CBSE 2020)

(7) The number of all possible matrices of order 2×3 with each entry 1 or 2 is (CBSE 2021)

(a) 16 (b) 6 (c) 64 (d) 24

(8) If a matrix A is both symmetric and skew symmetric then A is necessarily a

(a) Diagonal matrix (b) zero square matrix (c) square matrix (d) Identity matrix

(CBSE 2021)

(9) If $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of $ab - cd$ is (CBSE 2021)

(a) 4 (b) 16 (c) −4 (d) −16

(10) For the matrix $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $(X^2 - X)$ is (CBSE 2021)

(a) 2I (b) 3I (c) I (d) 5I

$5, \hat{i} < \hat{j}$
$3\hat{i} - 2\hat{j}, \hat{i} > \hat{j}$

The number of element in A which are more than 5 is

(a) 3 (b) 4 (c) 5 (d) 6 (CBSE 2021)

(12) For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, $P - Q$ is (CBSE 2021)

(a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 3 \\ 0 & -3 \\ -1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

13- If $\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$, then x equals

(A) 0 (B) −2 (C) −1 (D) 2 (CBSE - 2020 (D))

(14)- $A = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$, $X = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, $Y = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

then $AB + XY$ equals

(A) [28] (B) [24] (C) 28 (D) 24 CBSE(OD) - 2020

15- Suppose P and Q are two ~~diff~~ diff matrices of order $3 \times n$ and $n \times p$, then order of the matrix $P \times Q$ is

(A) $3 \times p$ (B) $p \times 3$ (C) $n \times n$ (D) 3×3

16 – If $|A| = |KA|$ where A is square matrix of order 2 then find sum of all possible values of K

17 -

Determinant

1. If A is a square matrix of order 3, such that $A(\text{Adj } A) = 10I$, then $|\text{Adj } A|$ is equal to (CBSE-2020)

(A) 1 (B) 10 (C) 100 (D) 101

2. If A is 3×3 matrix such that $|A| = 8$, then $|3A|$ equals (Delhi CBSE-2020)

(A) 8 (B) 24 (C) 72 (D) 216

3. If A is skew symmetric matrix of order 3, then the value of $|A|$ is CBSE (D)-2020

(A) 3 (B) 0 (C) 9 (D) 27

4. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then ~~the~~ value of x is. CBSE (OD)-2020

(A) 3 (B) 0 (C) -1 (D) 1

5. Let $A = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}$, and $B = \begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$, then $|AB|$ is $\boxed{\text{CBSE (OD)-2020}}$

(A) 460 (B) 2000 (C) 3000 (D) -7000

6. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ then Det (Adj A) equals

(A) a^{27} (B) a^{9} (C) a^{6} (D) a^{2}

7. If A is square matrix satisfying $A'A = I$, write the value of $|A|$

8. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$, find A^{-1}, hence solve the system of equations CBSE-2022

$$x + 3y + 4z = 8$$
$$2x + y + 2z = 5$$
$$5x + y + z = 7$$

(9) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} to solve the following equations

CBSE-2024

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

(10) $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ find AB and use it to solve the system of equations

$$x - 2y = 3$$
$$2x - y - z = 2$$
$$-2y + z = 3$$

CBSE-2023

(11) $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ prove that

CBSE-2023

$$f(\alpha)\, f(-\beta) = f(\alpha - \beta)$$

Chapter - 5 Continuity & Differentiability

1. If $\log(x^2+y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx} = \frac{x+y}{x-y}$

OR If $x^y - y^x = a^b$, find dy/dx [2019]

OR If $y = (\sin^{-1}x)^2$, Prove that $(1-x^2)y'' - xy' - 2 = 0$

2. If $x = \cos t + \log \tan \frac{t}{2}$, $y = \sin t$, then find $\frac{d^2y}{dx^2}$ at $x = \pi/4$ [2019]

3. ~~find: $\int \sin x \log \cos x \, dx$~~ if $y = x|x|$, find $\frac{dy}{dx}$ for $x < 0$ [2019]

4. if $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, PT $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$

OR if $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$ [2019]

5. if $(x-a)^2 + (y-b)^2 = c^2$, for some $c>0$, PT [2019]

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$ is a constant independent of a and b.

6. Differentiate $e^{\sqrt{3x}}$ w.r.t. x [2019]

OR if $y = \cos(\sqrt{3}x)$, then find $\frac{dy}{dx}$

7. If $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$, then prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$ [2019]

OR Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ w.r.t. x

8. If $\sin y = x\sin(a+y)$, then PT $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ [2019]

OR if $(\sin x)^y = x+y$, then find dy/dx

9. If $y = (\sec^{-1}x)^2$, $x>0$ then show that [2019]

$$x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3-x)\frac{dy}{dx} - 2 = 0$$

10. Find $\frac{dy}{dx}$, if $xy^2 - x^2 = 4$ [2019]

OR If $y = (\cot^{-1}x)^2$, show that $(x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$

11. If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ [2019]

OR find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right]$

, if $y = e^{x^2 \cos x} + (\cos x)^x$, then find $\left(\frac{dy}{dx}\right)$ [2020]

3) find the relationship between a and b so that the function 'f' defined by $f(x) = \begin{cases} ax+1 & \text{if } x \le 3 \\ bx+3 & \text{if } x > 3 \end{cases}$ is continuous at $x=3$ [2021]

OR check the differentiability of $f(x) = |x-3|$ at $x=3$.

(14) If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$ then show that $\frac{dy}{dx} = -x/y$ and hence show that

$$y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$ [2021]

OR

If $e^{y-x} = y^x$, Prove that $\frac{dy}{dx} = \frac{y(1+\log y)}{x \log y}$

(15) Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.

(16) Find the value of 'k' for which $f(x) = \begin{cases} 3x+5 & , x \ge 2 \\ kx^2 & , x < 2 \end{cases}$ is a continuous function [2021]

(17) if $y = (x + \sqrt{x^2-1})^2$, then show that $(x^2-1)\left(\frac{dy}{dx}\right)^2 = 4y^2$ [2023] [2023]

Chapter-6 Application of Derivatives

① A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs ₹ 70 per square meter for the base and ₹ 45 per square meter for the sides, what is the cost of least expensive tank? [2019]

2. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find the maximum volume of cone. [2019]

3. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume. [2019]

4. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of its edge is 12 cm? [2019]

5. Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base. [2019]

6. Find the local maxima and local minima, if any, of the following function. Also find the local maximum and the local minimum values, as the case may be: [2019]

$$f(x) = \sin x + \frac{1}{2}\cos 2x, \ 0 \le x \le \pi/2$$

7. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is (a) strictly increasing [2019]
(b) strictly decreasing

⑧ Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height 'h' and radius 'r' is one-third of the height of the cone, and the greatest volume of the cylinder is 4/9 times the volume of the cone. [2020]

9) Show that the function $f(x) = \frac{3}{x} + 7$ is strictly decreasing for $x \in R - \{0\}$

[2021]

10) find the interval to which 'a' belongs to if $f(x) = a(x - \cos x)$ is strictly decreasing in R. [2023]

11) Show that the function $f(x) = \frac{16 \sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

[2023]

Chapter - 7 Integrals

1) Find : $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$ [2019]

2) Find : $\int \sqrt{1 - \sin 2x}\, dx$, $\frac{\pi}{4} < x < \frac{\pi}{2}$ OR $\int \sin^{-1} 2x\, dx$ [2019]

OR find $\int \frac{3x+5}{x^2+3x-18} dx$ [2019]

OR Prove that: $\int_0^a f(x) dx = \int_0^a f(a-x) dx$, hence evaluate $\int_0^{\pi} \frac{x \sin x\, dx}{1+\cos^2 x}$ [2019]

3) find : $\int \frac{\tan^2 x \sec^2 x}{1 - \tan^6 x} dx$ [2019]

4) find : $\int \sin x \log \cos x\, dx$ [2019]

5) Evaluate: $\int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x\, dx$ [2019]

OR $\int_{-1}^{2} \frac{|x|}{x} dx$

6) Evaluate: $\int \frac{\cos x\, dx}{(1+\sin x)(2+\sin x)}$ OR $\int \sqrt{3 - 2x - x^2}\, dx$ [2019]

7) find : $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$ OR $\int \frac{(x-3)e^x}{(x-1)^3} dx$ [2019]

8) find : $\int \frac{(x^2+x+1)}{(x+2)(x^2+1)} dx$ OR PT $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

and hence evaluate $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$ [2019]

9) Find : $\int \sqrt{3 - 2x - x^2}\, dx$ [2019]

10) find : $\int \frac{(x-5)}{(x-3)^3} e^x dx$ [2019]

11) Find : $\int \frac{2\cos x\, dx}{(1-\sin x)(2-\cos^2 x)}$ [2019]

(12) Find: $\int \frac{\sin x - \cos x}{\sqrt{1+\sin 2x}}\, dx$, $0 < x < \pi/2$ [2019]

OR Find $\int \frac{\sin(x-a)}{\sin(x+a)}\, dx$

OR Find $\int (\log x)^2\, dx$

(13) Find: $\int \frac{\sin 2x\, dx}{(\sin^2 x+1)(\sin^2 x+3)}$ [2019]

OR PT $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ and hence evaluate [2019]

$\int_{\pi/6}^{\pi/3} \frac{dx}{\sqrt{\tan x}+1}$

(14) Evaluate: $\int_{-\frac{\pi}{4}}^{\pi/4} \sec^2 x\, dx$ [2020]

(15) Find: $\int x^4 \log x\, dx$ OR $\int \frac{2x}{\sqrt[3]{x^2+1}}\, dx$ [2020]

(16) Evaluate: $\int_1^3 |2x-1|\, dx$ [2020]

(17) If $x = at^2$, $y = 2at$, then find $\frac{d^2y}{dx^2}$ [2020]

(18) if $y = e^{x^2\cos x} + (\cos x)^x$, then find $\frac{dy}{dx}$ [2020]

(19) find: $\int \sec^3 x\, dx$ [2020]

(20) find: $\int e^x \left(\log\sqrt{x} + \frac{1}{2x}\right) dx$ OR $\int e^{2\log x}\, dx$ [2021]

(21) Find: $\int \frac{x^2+2}{x^2+1}\, dx$ OR $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ [2021]

(22) find: $\int \frac{x^2\,dx}{(x-1)(x+1)^2}$ [2021]

(23) If $\frac{d}{dx}[F(x)] = \frac{\sec^4 x}{\operatorname{cosec}^4 x}$ and $F(\pi/4) = \pi/4$, then find $F(x)$ [2022]

OR find: $\int \frac{\log x - 3}{(\log x)^4}\, dx$

(24) Find: $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$ OR $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(4+\sin x)}\, dx$ [2022]

(25) Evaluate: $\int_0^{\pi} \frac{x}{1+\sin x}\, dx$ [2022]

(26) Evaluate: $\int_0^{\pi/6} \sec^2(x - \pi/6)\, dx$ [2023]

27 Evaluate: $\int_0^{\pi/2} [\log(\sin x) - \log(\cos x)]\, dx$ [2023]

OR

$$\int \frac{dx}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}$$

28 Evaluate: $\int_0^{\pi/2} e^x \sin x\, dx$ OR $\int \frac{dx}{\cos(x-a)\cos(x-b)}$ [2023]

Chapter 8 Application of Integrals

① Using method of integration, find the area of the region enclosed between two circles $x^2+y^2=4$ and $(x-2)^2+y^2=4$ [2019]

② Find the area of the region
$\{(x,y): 0 \le y \le x^2, \; 0 \le y \le x+2, \; -1 \le x \le 3\}$
[2019]

③ using integration, find the area of the region bounded by the triangle whose vertices are $(2,-2)$, $(4,5)$ and $(6,2)$. [2020]

④ using integration, find the area of the region bounded by the curve $y^2=4x$, y axis and $y=3$ [2021]

OR using integration, find the area of the region bounded by the line $2y=-x+8$, x-axis, $x=2$ and $x=4$

⑤ using integration, find the area bounded by the circle $x^2+y^2=9$
[2021]

⑥ using integration, find the area of the region $\{(x,y): 4x^2+9y^2 \le 36, \; 2x+3y \ge 6\}$
[2022]

OR using integration, find the area of the region bounded by lines $x-y+1=0$, $x=-2$, $x=3$ and x-axis. [2022]

⑦ Find the area of the region bounded by the curves $x^2=y$, $y=x+2$ and x-axis, using integration.
[2023]

L.P.P.

Previous year's Questions

CLASS – XII

1) The graph of the inequality $2x+3y>6$ is (2020 AI) Delhi
   a) half plane that contains the origin (b) entire XOY Plane
   c) half plane that neither contains the origin nor the points of the line $2x+3y=6$
   d) Whole XOY-plane excluding the points on the line $2x+3y=6$.

2) Solve the foll. L.P.P. graphically (2023 Main)

$$\text{Min. } Z = 60x + 80y$$

Subject to constraints: $3x+4y \geq 8$, $5x+2y \geq 11$, $x, y \geq 0$

3) The sol. set of the inequation $3x+5y<7$ is
   a) Whole xy-plane except the points lying on the line $3x+5y=7$.
   b) Whole xy-plane along with the points lying on the line $3x+5y=7$
   c) Open half plane containing the origin except the points of line $3x+5y=7$.
   d) Open half plane not containing the origin.

4) Which of the following points satisfies both the inequations $2x+y \leq 10$, and $x+2y \geq 8$?
   a) $(-2,8)$ (b) $(3,2)$ (c) $(-5,6)$ (d) $(4,2)$

5) Solve the following L.P.P. graphically:
   Max. $Z = 5x+3y$ subject to the constraints
   $3x+5y \leq 15$, $5x+2y \leq 10$, $x, y \geq 0$.

6) Solve the following L.P.P. graphically:
   Min. $Z = x+2y$ subject to the constraints
   $2x+y \geq 3$, $x+2y \geq 6$, $x \geq 0$, $y \geq 0$.

7) The corner points of the feasible region in the graphical representation of a L.P.P. are $(2,72)$, $(15,20)$ and $(40,15)$. If $Z = 18x+9y$ be the objective function, then:
   a) Z is max. at $(2,72)$, min. at $(15,20)$
   b) Z is max. at $(15,20)$, min. at $(40,15)$
   c) Z is max. at $(40,15)$, min. at $(15,20)$
   d) Z is max. at $(40,15)$, min. at $(2,72)$

8) The number of corner points of the feasible region determined by the constraints $x-y \geq 0$, $2y \leq x+2$, $x \geq 0$, $y \geq 0$ is
   (a) 2 (b) 3 (c) 4 (d) 5

9) Solve graphically the following L.P.P.: max. $Z = 6x+3y$
   subject to the constraints $4x+y \geq 80$, $3x+2y \leq 150$, $x+5y \geq 115$, $x \geq 0$, $y \geq 0$.

10) Solve the following L.P.P. graphically: (2023 C)
    Min. $Z = 6x+7y$ subject to the constraints $2x+y \geq 8$, $x+2y \geq 10$, $x, y \geq 0$.

11) Solve the following L.P.P. graphically
    Max. $Z = 10x+15y$ subject to the constraints:
    $3x+2y \leq 50$, $x+4y \geq 20$, $x \geq 8$, $y \geq 0$.

12) Solve the following L.P.P. graphically:
    Min. $Z = 3x+8y$ subject to the constraints
    $3x+4y \geq 8$, $5x+2y \geq 11$, $x \geq 0$, $y \geq 0$.

Differential Equation

Previous year's Questions

CLASS – XII

① Solve the following diff. eq. $\frac{dy}{dx}+y=\cos x-\sin x$ (2019 Abroad)

② Solve the diff. eq. $\frac{dy}{dx}=1+x^2+y^2+x^2y^2$, given that $y=1$ when $x=0$

③ find the particular sol. of the diff. eq. $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$, given that $y=1$ when $x=0$.

④ Write the order and the degree of the following diff. equ.: (2019 Delhi)

$$x^2\left(\frac{d^2y}{dx^2}\right)^2+x\left(\frac{dy}{dx}\right)^4=0$$

(5) Solve the diff. equ. $\frac{dy}{dx}-\frac{2x}{1+x^2}y=x^2+2$

(6) Solve the diff. equ. $(x+1)\frac{dy}{dx}=2e^{-y}-1,\ y(0)=0$.

(7) Find the order and the degree of the diff. equ. $x^2\frac{d^2y}{dx^2}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^4$

(8) Sol. the diff. equ. $x\,dy-y\,dx=\sqrt{x^2+y^2}\,dx$, given that $y=0$ when $x=1$.

(9) Sol. the diff. equ. $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$, subject to the initial cond. $y(0)=0$.

(10) Write the order & degree of the diff. eq. $\left(\frac{d^4y}{dx^4}\right)^2=\left[x+\left(\frac{dy}{dx}\right)^2\right]^3$ (2019 AI)

(11) Solve the diff. eq. $\frac{dy}{dx}=\frac{x+y}{x-y}$

(12) Solve the diff. eq. $(1+x^2)dy+2xy\,dx=\cot x\,dx$.

(13) Sol. the diff. equ. $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)=0$, given $x=1,\ y=\frac{\pi}{2}$ (2020 AI)

(14) find integrating factor of the diff. equ. $x\frac{dy}{dx}+2y=x^2$. (2020 Del.)

(15) Find the gen. sol. of the diff. eq. $y e^{x/y}dx=(xe^{x/y}+y^2)dy,\ y\neq 0$.

(16) Find the gen. sol. of the foll. diff. equ. $\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$. (2022 AI)

(17) Solve the foll. diff. eq. $(y-\sin^2x)dx+\tan x\,dy=0$.

(18) Find the gen. sol. of the diff. equ. $(x^3+y^3)dy=x^2y\,dx$.

(19) Find the gen. sol. of the diff. eq. $\sec^2x\cdot\tan y\,dx+\sec^2y\cdot\tan x\,dy=0$

(20) Find the P. sol. of the diff. equ $x\frac{dy}{dx}+y+\frac{1}{1+x^2}=0$, given that $y(1)=0$.

(21) Find the gen. sol. of the diff. equ. $x(y^3+x^3)dy=(2y^4+5x^3y)dx$. (2023) AI

(22) The order and the degree of the diff. eq. $\left(1+3\frac{dy}{dx}\right)^2=4\frac{d^3y}{dx^3}$

(23) The order and degree (if defined) of the diff. equ $\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^3=x\sin\left(\frac{dy}{dx}\right)$

(24) The I.F. for sol. the diff. equ. $x\frac{dy}{dx}-y=2x^2$.

(25) Find the Particular sol. of the diff. equ. $\frac{dy}{dx}=\frac{x+y}{x},\ y(1)=0$.

(26) Find the gen. sol. of the diff. equ. $e^x\tan y\,dx+(1-e^x)\sec^2y\,dy=0$.

(27) The sol. of the diff. equ. $\frac{dx}{x}+\frac{dy}{y}=0$

(28) What is the product of the order & degree of the diff. equ.

$$\frac{d^2y}{dx^2}\sin y+\left(\frac{dy}{dx}\right)^3\cos y=\sqrt{y}\ ?$$

(29) find the gen. sol. of the diff. equ. $\frac{d}{dx}(xy^2)=2y(1+x^2)$

(30) Solve the foll. diff. equ. $xe^{y/x}-y+x\frac{dy}{dx}=0$.

(1) The value of P for which the vectors $2\hat{i}+P\hat{j}+\hat{k}$ and $-4\hat{i}-6\hat{j}+26\hat{k}$ are Perpendicular to each other is

(a) 3 (b) -3 (c) $-\frac{17}{3}$ (d) $\frac{17}{3}$

(2) $(\hat{i}\times\hat{j})\cdot\hat{j}+(\hat{j}\times\hat{i})\cdot\hat{k}$ is

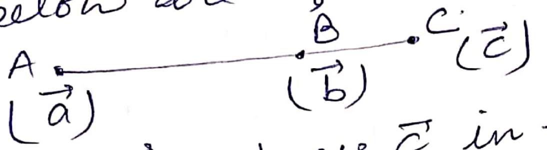
(a) 2 (b) 0 (c) 1 (d) -1

(3) If $\vec{a}+\vec{b}=\hat{i}$ and $\vec{a}=2\hat{i}-2\hat{j}+2\hat{k}$, the $|\vec{b}|$ equals:

(a) $\sqrt{14}$ (b) 3 (c) $\sqrt{12}$ (d) $\sqrt{17}$

(4) Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i}+\hat{j}+\hat{k}$.

(5) Position vectors of the points A, B and C as shown in fig. below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.



If $\vec{AC}=\frac{5}{4}\vec{AB}$, express $\vec{c}$ in terms of $\vec{a}$ & $\vec{b}$.

~~(6) The value of P for which the vectors $2\hat{i}+P\hat{j}+\hat{k}$ and $-4\hat{i}-6\hat{j}+26\hat{k}$ are perpendicular to each other, is~~

(6) If the vector $\hat{i}-b\hat{j}+\hat{k}$ is equally inclined to the coordinate axes, then the value of b is

(a) -1 (b) 1 (c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$

(7) If $\vec{a}+\vec{b}=\hat{i}$ and $\vec{a}=2\hat{i}-2\hat{j}+2\hat{k}$, then $|\vec{b}|$ equals

(a) $\sqrt{14}$ (b) 3 (c) $\sqrt{12}$ (d) $\sqrt{17}$

(8) For two non-zero vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a}-\vec{b}|=|\vec{a}+\vec{b}|$, then find the angle between $\vec{a}$ & $\vec{b}$.

(9) For what value of λ, the projection of vector $\hat{i}+\lambda\hat{j}$ on vector $\hat{i}-\hat{j}$ is $\sqrt{2}$?

(a) -1 (b) 1 (c) 0 (d) 3

(10) Unit vector along $\vec{PQ}$, where coordinates of P and Q respectively are $(2,1,-1)$ and $(4,4,-7)$ is

(a) $2\hat{i}+3\hat{j}-6\hat{k}$ (b) $-2\hat{i}-3\hat{j}+6\hat{k}$ (c) $-\frac{2\hat{i}}{7}-\frac{3\hat{j}}{7}+\frac{6\hat{k}}{7}$ (d) $\frac{2\hat{i}}{7}+\frac{3\hat{j}}{7}-\frac{6\hat{k}}{7}$

11) Position vector of the mid-point of line segment AB is $3\hat{i}+2\hat{j}-3\hat{k}$. If position vector of the point A is $2\hat{i}+3\hat{j}-4\hat{k}$, then position vector of the point B is

a) $\frac{5\hat{i}}{2}+\frac{5}{2}\hat{j}-\frac{7}{2}\hat{k}$ b) $4\hat{i}+\hat{j}-2\hat{k}$ c) $5\hat{i}+5\hat{j}-7\hat{k}$ d) $\frac{\hat{i}}{2}-\frac{\hat{j}}{2}+\frac{\hat{k}}{2}$

12) Projection of vectors $2\hat{i}+3\hat{j}$ on the vector $3\hat{i}-2\hat{j}$ is

a) 0 b) 12 c) $\frac{12}{\sqrt{13}}$ d) $\frac{-12}{\sqrt{13}}$

13) If $\vec{a},\vec{b},\vec{c}$ are three non-zero unequal vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$, then find the angle between $\vec{a}$ and $\vec{b}-\vec{c}$.

14) Unit vector along $\vec{PQ}$, where co-ordinates of P and Q respectively are (2,1,−1) and (4,4,−7) is

a) $2\hat{i}+3\hat{j}-6\hat{k}$ b) $-2\hat{i}-3\hat{j}+6\hat{k}$ c) $-\frac{2}{7}\hat{i}-\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$ d) $\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}$

15) If in ΔABC, $\vec{BA}=2\vec{a}$ and $\vec{BC}=3\vec{b}$, then $\vec{AC}$ is

a) $2\vec{a}+3\vec{b}$ b) $2\vec{a}-3\vec{b}$ c) $3\vec{b}-2\vec{a}$ d) $-2\vec{a}-3\vec{b}$

16) If $|\vec{a}\times\vec{b}|=\sqrt{3}$ and $\vec{a}\cdot\vec{b}=-3$, then angle between $\vec{a}$ & $\vec{b}$ is

a) $\frac{2\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{3}$ d) $\frac{5\pi}{6}$

17) If the angle between $\vec{a}$ & $\vec{b}$ is $\frac{\pi}{3}$ and $|\vec{a}\times\vec{b}|=3\sqrt{3}$, then the value of $\vec{a}\cdot\vec{b}$ is

a) 9 b) 3 c) $\frac{1}{9}$ d) $\frac{1}{3}$

18) The position vector of three consecutive vertices of a ||gm ABCD are $A(4\hat{i}+2\hat{j}-6\hat{k})$, $B(5\hat{i}-3\hat{j}+\hat{k})$ and $C(12\hat{i}+4\hat{j}+5\hat{k})$. The P.V. of D is given by

a) $-3\hat{i}-5\hat{j}-10\hat{k}$ b) $21\hat{i}+3\hat{j}$ c) $11\hat{i}+9\hat{j}-2\hat{k}$ d) $-11\hat{i}-9\hat{j}+2\hat{k}$

19) If points A, B and C have P.V. $2\hat{i}$, $\hat{j}$ & $2\hat{k}$ respectively, then show that ΔABC is an isosceles triangle.

20) Find the equations of the diagonals of the ||gm PQRS whose vertices are P(4,2,−6), Q(5,−3,1), R(12,4,5) and S(11,9,−2). Use these equations to find the point of intersection of diagonals.

21) If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is.

a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{2}$ d) 0

22) If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is

a) $\pi/2$ b) π c) $\pi/4$ d) 0

23) In ΔABC, $\vec{AB}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{AC}=3\hat{i}-\hat{j}+4\hat{k}$. If D is mid point of BC, then vector $\vec{AD}$ is equal to

a) $4\hat{i}+6\hat{k}$ b) $2\hat{i}-2\hat{j}+2\hat{k}$ c) $\hat{i}-\hat{j}+\hat{k}$ d) $2\hat{i}+3\hat{k}$

Vector
(2023) (2)

$$\text{If } \vec{r} = 3\hat{i}-2\hat{j}+6\hat{k}, \text{ find the value of } (\vec{r}\times\hat{j})\cdot(\vec{r}\times\hat{k})-12.$$

25) If the angle between the vectors $\vec{a}$ & $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{a}\times\vec{b}|=1$, then $\vec{a}\cdot\vec{b}$ is equal to

a) -1 b) 1 c) $\frac{1}{\sqrt{2}}$ d) $\sqrt{2}$

26) A unit vector along the vector $4\hat{i}-3\hat{k}$ is:

a) $\frac{1}{7}(4\hat{i}-3\hat{k})$ b) $\frac{1}{5}(4\hat{i}-3\hat{k})$ c) $\frac{1}{\sqrt{7}}(4\hat{i}-3\hat{k})$ d) $\frac{1}{\sqrt{5}}(4\hat{i}-3\hat{k})$

27) If θ is the angle between two vectors $\vec{a}$ & $\vec{b}$, then $\vec{a}\cdot\vec{b}\geq 0$ only when a) $0<\theta<\pi/2$ b) $0\leq\theta\leq\frac{\pi}{2}$ c) $0<\theta<\pi$ d) $0\leq\theta\leq\pi$

28) If the projection of the vector $\hat{i}+\hat{j}+\hat{k}$ on the vector $p\hat{i}+\hat{j}-2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.

29) If $\vec{a}=4\hat{i}-\hat{j}+\hat{k}$ & $\vec{b}=2\hat{i}-2\hat{j}+\hat{k}$, then find a unit vector along the vector $\vec{a}\times\vec{b}$.

30) The sine of the angle between the vectors $\vec{a}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}+2\hat{k}$ is:

a) $\sqrt{\frac{5}{21}}$ b) $\frac{5}{\sqrt{21}}$ c) $\sqrt{\frac{3}{21}}$ d) $\frac{4}{\sqrt{21}}$

31) Two vectors $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ & $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ are collinear if a) $a_1b_1+a_2b_2+a_3b_3=0$ (b) $\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}$

c) $a_1=b_1, a_2=b_2, a_3=b_3$ (d) $a_1+a_2+a_3=b_1+b_2+b_3$

32) The magnitude of the vector $6\hat{i}-2\hat{j}+3\hat{k}$ is

a) 1 b) 5 c) 7 d) 12

33) If the vectors $\vec{a}$ & $\vec{b}$ are such that $|\vec{a}|=3$, $|\vec{b}|=\frac{2}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector, then find the angle between $\vec{a}$ & $\vec{b}$.

34) Find the area of a ||gm whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ & $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

35) Three vectors $\vec{a}, \vec{b}$ & $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, if $|\vec{a}|=3$, $|\vec{b}|=4$ & $|\vec{c}|=2$.

(2022)

36) If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{a}\cdot\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then find $|\vec{b}|$.

37) If $\vec{a}$ & $\vec{b}$ are two vectors such that $|\vec{a}+\vec{b}|=|\vec{b}|$, then prove that $(\vec{a}+2\vec{b})$ is $\perp$ to $\vec{a}$.

38) If $\vec{a}$ & $\vec{b}$ are unit vectors and θ is the angle between them then prove that $\sin\frac{\theta}{2}=\frac{1}{2}|\vec{a}-\vec{b}|$

39) If $\hat{i},\hat{j},\hat{k}$ are unit vectors along three mutually perpendicular directions, then

a) $\hat{i}\cdot\hat{j}=1$ b) $\hat{i}\times\hat{j}=1$ c) $\hat{i}\cdot\hat{k}=0$ d) $\hat{i}\times\hat{k}=0$

40) In a $\parallel^{gm}$ PQRS, $\vec{PQ} = 3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{PS} = -\hat{i}-2\hat{k}$. find $|\vec{PR}|$ and $|\vec{QS}|$. (2020)

41) If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are four non-zero vectors such that $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = 4\vec{b}\times\vec{d}$, then show that $(\vec{a}-2\vec{d})$ is parallel to $(2\vec{b}-\vec{c})$ where $\vec{a} \neq 2\vec{d}, \vec{c} \neq 2\vec{b}$.

42) the two adjacent sides of a parallelogram are represented by $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. find the unit vectors parallel to its diagonals. Using the diagonals vectors, find the area of the parallelogram also. (2019)

43) X and Y are two P.V. $3\vec{a}+\vec{b}$ & $\vec{a}-3\vec{b}$ respectively. Write the P.V. of a point Z which divides the line segment XY in the ratio 2:1 externally.

44) Let $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$ & $\vec{b} = 3\hat{i}-\hat{j}+2\hat{k}$ be two vectors. Show that the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are $\perp$ to each other.

45) If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively are the P.V. of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\vec{AB}$ and $\vec{CD}$ are collinear or not.

46) Find a unit vector $\perp$ to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i}-7\hat{j}+7\hat{k}$ & $\vec{b} = 3\hat{i}-2\hat{j}+2\hat{k}$.

47) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three ~~non~~ vectors such that $|\vec{a}|=1, |\vec{b}|=2$ and $|\vec{c}|=3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$, and $\vec{b}, \vec{c}$ are $\perp$ to each other, then find $|3\vec{a}-2\vec{b}+2\vec{c}|$.

IPR
2020

Three Dimensional Geometry Class 12 Important Questions Previous Year Questions

1. If a line makes angles 90°, 135°, 45° with the x, y and z axes respectively, find its direction cosines.
2. Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.
3. What are the direction cosines of a line which makes equal angles with the coordinate axes?
4.A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.
5. The equations of a line is $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.
6. If a line makes angles 90° and 60°, respectively with the positive directions of X and Y-axes, find the angle which it makes with the positive direction of Z-axis.
7. The equations of a line is $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.
8. If a line makes angles a,p,y with the position direction of coordinate axes, then write the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$. 11

9. Write the distance of a point P(a, b, c) from X-axis.

10. If the cartesian equation of a line is 3−x5=y+47=2z−64 then write the vector equation for the line.

11. Write the equation of the straight line through the point (α β γ) and parallel to Z-axis.

12. Find the direction cosines of the line$(4-x)/2 = y/6 = (1-z)/3$

13. Write the vector equation of a line passing through point (1, − 1, 2) and parallel to the line whose equation is$(x-3)/1 = (y-1)/2 = (z+1)/-2$.
14. Find the cartesian equation of the line which passes through the point (-2, 4, − 5) and is parallel to the line $(x+3)/3 = (4-y)/5 = (z+8)/6$
15. If a line has direction ratios (2, − 1, − 2), then what are its direction cosines?

16. Write the vector equation of the line given by $(x-5)/3 = (y+4)/7 = (z-6)/2$
17. Equation of line is $(4-x)/2 = (y+3)/2 = (z+2)/1$
Find the direction cosines of a line parallel to above line.

18. The cartesian equation of a line is $6x - 2 = 3y + 1 = 2z - 2$. Find the direction cosines of the line. Write down the cartesian and vector equations of a line passing through (2, − 1, − 1) which are parallel to the given line.

19. If the equations of line AB is $3-x/1 = y+2/-2 = z-5/4$, then write the direction ratios of the line parallel to above line AB.

20. Find the distance of point (2, 3, 4) from X-axis.

21. Write the vector equation of the following $x-5/3 = y+4/7 = 6-z/2$

22. Find the vector equation of the line passing through the point A (1, 2, − 1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

23. The x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, − 2) is 4. Find its z-coordinate.

24. Find the value of λ, so that the lines $1-x3=7y-14\lambda=z-32$ and $7-7x3\lambda=y-51=6-z5$ are at right angles. Also, find whether the lines are intersecting or not.

25. If the lines $x-1/-3=y-2/2\lambda=z-3/2$ and $x-1/3\lambda=y-1/2=z-6/-5$ are perpendicular, find the value of λ. Hence find whether the lines are intersecting or not.

26. Find the shortest distance between the lines $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$.

27. Find the shortest distance between the lines $x-1/2 = y-2/3 = z-3/4$ and $x-2/3 = y-4/4 = z-5/5$.

28. Find the vector and cartesian equations of the line through the point (1, 2, − 4) and perpendicular to the two lines
$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and
$\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

29. Find the equation of a line passing through the point (1, 2, − 4) and perpendicular to two lines
$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and
$\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

30. Find the coordinates of the foot of perpendicular drawn from the point A (- 1, 8, 4) to the line joining the points B(0, − 1, 3) and C(2, − 3, − 1). Hence, find the image of the point A in the line BC.

31. Prove that the line through A (0, − 1, − 1) and B(4, 5, 1) intersects the line through C (3, 9, 4) and D (- 4, 4, 4).

32. Show that the lines
$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and
$\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also, find their point of intersection.

33. Find the direction cosines of the line $x+22=2y-76=5-z6$. Also, find the vector equation of the line through the point A(-1, 2, 3) and parallel to the given line.

34. Find the angle between the lines
$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$
and $\vec{r} = 7\hat{i} - 6\hat{j} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$.

35. Show that the lines = $x+1/3 = y+3/5 = z+5/7$ and $x-2/1 = y-4/3 = z-6/5$ intersect. Also, find their point of intersection.

36. Find the value of p, so that the lines

l_1: $1-x/3 = 7y-1/4p = z-3/2$ and

l_2: $7-7x/3p = y-5/1 = 6-z/5$ are

perpendicular to each other. Also, find the equation of a line passing through a point (3, 2, − 4) and parallel to line l_1.

37. A line passes through the point (2, − 1, 3) and is perpendicular to the lines

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda\,(2\hat{i} - 2\hat{j} + \hat{k})$

and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$.

Obtain its equation in vector and cartesian forms.

38. Find the shortest distance between the two lines whose vector equations are

$\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda\,(\hat{i} - 3\hat{j} + 2\hat{k})$ and

$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$.

39. Find the shortest distance between the following lines.

$x-3/1 = y-5/-2 = z-7/1$, $x+1/7 = y+1/-6 = z+1/1$

40. Find the distance between the lines l_1 and l_2 given by

l_1:$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda\,(2\hat{i} + 3\hat{j} + 6\hat{k})$

l_2:$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$.

PROBABILITY

1. A bag contains 3 red and 4 white balls. Three balls are drawn at random, one-by-one without replacement from the bag. If the first ball drawn is red in colour, then find the probability that the remaining two balls drawn are also red in colour.

2. A coin is tossed twice. The following table shows the probability distribution of number of tails :

X	0	1	2
P(X)	K	6K	9K

(a) Find the value of K.

(b) Is the coin tossed biased or unbiased ? Justify your answer.

3. There are two bags. Bag I contains 1 red and 3 white balls, and Bag II contains 3 red and 5 white balls. A bag is selected at random and a ball is drawn from it. Find the probability that the ball so drawn is red in colour.

4. Three friends A, B and C got their photograph clicked. Find the probability that B is standing at the central position, given that A is standing at the left corner.

5. Assertion (A) : Given two independent events A and B such that P(A) = 0·3 and P(B) = 0·6, then

P(A and not B) = 0·12.

Reason (R) : For two independent events A and B,

P(A and B) = P(A) . P(B).

6. Let A and B be the events such that P(A) =1/ 2

, P(B) = 7/12 and P(not A or not B) = 1/4. Find whether A and B are (i) mutually exclusive (ii) independent.

7. Find the mean of the number of tails in two tosses of a coin .

8. A shopkeeper sells three types of flower seeds A1, A2 and A3. They are sold as a mixture where the proportions are 4 : 4 : 2 respectively. The germination rates of the three types of seeds

are 45%, 60% and 35%. Based on the above information, answer the following questions :

(i) What is the probability of a randomly chosen seed to germinate ?

(ii) What is the probability that the randomly selected seed is of type A1, given that it germinates ?

9. A fair die is rolled. Events E and F are E = {1, 3, 5} and F = {2, 3} Find the p(E/F)

1) 2/3 2) 1/3 3)1/6 4) ½

10. Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

11. Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without

replacement). Find the probability distribution of number of

selected persons who always speak the truth.

12. Case study In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions :

(i) Find the probability that the age of the selected student is a composite number.

(ii) Let X be the age of the selected student. What can be the value of X ?

(iii) (a) Find the probability distribution of random variable X and hence find the mean age.

(iii) (b) A student was selected at random and his age was found to be greater tha).n 15 years. Find the probability that his age is a prime number.

13 . Ashima can hit a target 2 out of 3 times. She tried to hit the target twice. The probability that she missed the target exactly once is

(a)2/ 3 (b 1/) 3 (c) 1/9 (d)4/ 9

14. The probability distribution of a random variable X is given below :

X	1.	2.	3
P(X)	k/2.	K/3.	K/6

(i) Find the value of k.

(ii) Find P(1 <_X< 3

(iii) Find E(X), the mean of X.

15. . (a) Let A and B be the events such that P(A) = ½

, P(B) = 7/12 and P(not A or not B) =1/ 4

Find whether A and B are (i) mutually exclusive

(ii) independent.

16. Find the mean of the number of tails in two tosses of a Coin.

17. The probability distribution of a random variable X is given below :

X 1 2. 3

P(X) k/2 3/k. 6/k

(i) Find the value of k.

(ii) Find P(1< X < 3).

(iii) Find E(X), the mean of X.

18. A and B are independent events such that P(A B) = ¼

And (A. B) = 1/6 Find P(A) and P(B)

19. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below : Types of Yoga

Hatha Yoga

Bikram Yoga

Vinyasa Yoga

Kundalini Yoga

Anusara Yoga

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0·44.

On the basis of the above information, answer the following questions :

(i) Find the value of x.

(ii) Find the value of y.

(iii) (a) Find P(C/B) vein diagrams 65/1/1

OR(iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

20. Two events A and B will be independent, if :

(a) A and B are mutually exclusive

(b) P(A) = P(B)

(c) P(A' B') = [1- P(A)] [1- P(B)]

(d) P(A) + P(B) = 1

21 . Read the following passage and answer the questions given below :

There are ten cards numbered 1 to 10 and they are placed in a box and then mixed up thoroughly. Then one card is drawn at random from the box.

Based on the above, answer the following questions :

(i) What is the probability that the number on the drawn card is greater than 4 ?

(ii) If it is known that the number on the drawn card is

greater than 4, then what is the probability that it is an

even number ?

22. The events E and F are independent. If P(E) = 0·3 and P(E F) = 0·5, then P(E/F) P(F/E) equals :

(a) 1/7 (b) 2/7 (c) 3/ 35 (d)1/ 70

23. From a lot of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.

24 . A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let : E1: represent the event when many workers were not present for the job;

E2 : represent the event when all workers were present;

And E : represent completing the construction work on time. Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job ?

(ii) What is the probability that construction will be completed on time ?

(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ?

OR(iii) (b) What is the probability that all workers were present given that the construction job was completed on time ?

25. For two events A and B, if P(A) = 0·4, P(B) = 0·8 and P(B/A) = 0·6, then P(AU B) is :

(a) 0·24 (b) 0·3 (c) 0·48 (d) 0·96

(b) Two fair dice are thrown simultaneously. If X denotes the number of sixes, find the mean of X.

26. A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

27. There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

28.. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.

29. The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability

distribution of X.

X :	1	2.	3	4	5	6.	7.	8
P(X) :	p	2p	2p	p	2p	p^2	2p^2	7p^2+p

Based on the above information, answer the following questions :

(i) Find the value of p.

(ii) Find P(X> 6).

(iii) (a) Find P(X = 3m), where m is a natural number.

OR

(iii) (b) Find the mean E(X).

30. If P(A n B) =1/ 8 and P(not A) = ¾, then P(B/A) is equal to :

(a) ½ (b1/ 3(c)1/ 6 (d) 2/3

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