**Quadratic Equations**

**YEAR - 2019**

**SECTION - A**

Q1- Find the nature of roots of the quadratic equation \(2x^2 - 4x + 3 = 0\).

Q2- For what values of \(k\), the roots of the equation \(x^2 + 4x + k = 0\) are real?

Q3 - Write the discriminant of the quadratic equation \((x + 5)^2 = 2(5x - 3)\).

Q For what values of \(k\) does the quadratic equation \(4x^2 - 12x - k = 0\) have no real roots?

Or

Find the value of \(k\) for which the roots of the equation \(3x^2 - 10x + k = 0\) are reciprocal of each other.

Q Find the value of \(k\) for which \(x = 2\) is a solution of the equation \(kx^2 + 2x - 3 = 0\).

Or

Find the value/s of \(k\) for which the quadratic equation \(3x^2 + kx + 3 = 0\) has real and equal roots.

**SECTION - B**

Q3- Find the value(s) of \(k\) so that the pair of equations \(x + 2y = 5\) and \(3x + ky + 15 = 0\) has a unique solution.

Q4 - Show that every positive odd integer is of the form \((4q+1)\) or \((4q+3)\), where \(q\) is some integer.

Q5- Find \(e\) if the system of equations \(ex + 3y + (3 - e) = 0; 12x + cy - e = 0\) has infinitely many solutions.

Q Using completing the square method, show that the equation \(x^2 - 8x + 18 = 0\) has no solution.

Q Find the value(s) of \(k\) for which the pair of equations \(kx + 2y = 3; 3x + 6y = 10\) has a unique solution.

Q For what value of \(k\), does the system of linear equations \(2x + 3y = 7; (k - 1) x + (k + 2) y = 3k\) have an infinite number of solutions?

**SECTION - C**

Q3- For what value of \(k\), is the polynomial \(f(x) = 3x^4 - 9x^3 + x^2 + 15x + k\) completely divisible by \(3x^2 - 5\)?

Q - Find the value of \(k\) such that the polynomial \(x^2 - (k+6)x + 2(2k-1)\) has sum of its zeros equal to half of their product.

Q A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

Q4- Write all the values of \(p\) for which the quadratic equation \(x^2 + px + 16 = 0\) has equal roots. Find the roots of the equation so obtained.

**SECTION - D**

Q- Two water taps together can fill a tank in 17 hours. The tap with longer 8 diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

Or

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Q5- In a class test, the sum of Arun’s marks in Hindi and English is 30. Had he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

Q A pole has to be erected at a point on the boundary of a circular park of diameter 13 m in such a way that the...
difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 m. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

**Year 2020**

**SECTION A**

Q1-If one of the zeroes of the quadratic polynomial $x^2+3x+k$ is 2, then the value of $k$ is
(a) 108  (b) -10  (c) -7  (d) -21

Q2-The quadratic polynomial, the sum of whose zeroes is -5 and their product is 5, is
(a) $x^2+5x+6$  (b) $x^2-5x+6$  (c) $x^2-5x-6$  (d) $x^2+5x+6$

Q3-The value of $k$ for which the system of equations $x+y-4=0$ and $2x+ky=3$, has no solution, is
(a) -2  (b) #2  (c) 3  (d) 2

Q5- the roots of the quadratic equation $x^2 - 0.04 = 0$ are
(a) + 0.2  (b) + 0.02  (c) 0.4  (d) 2

Q 6 -The quadratic equation $x^2-4x + k = 0$ has distinct real roots if
(A) $k = 4(B)k > 4(C)k = 16(D)k < 4$

Q 7 - The value(s) of $k$ for which the quadratic equation $2x^2+ kx + 2 = 0$ has equal roots, is
(A)4(B)•4(C)–4(D)0

**SECTION-B**

Q 8-Solve for $x$ :
$6x^2+ 11x + 3 = 0$

Q9-The present age of a father is three years more than three times the age of his son. Three years hence the father’s age will be 10 years more than twice the age of the son. Determine their present ages.

**SECTION-C**

Q 10 -Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a +0, c +0$.

Or

Q11-Divide the polynomial $f(x)=3x^2-x^3-3x+5$ by the polynomial $g(x)=x-1-x^2$ and verify the division algorithm. This

Q12- Find the values of $k$, for which the quadratic equation $(k + 4) x^2 + (k + 1) x + 1 = 0$ has equal roots

Q13-On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$ respectively. Find $g(x)$.

**SECTION – D**
Q14-Show that the square of any positive integer cannot be of the form \((5q+2)\) or \((5q+3)\) for any integer \(q\).

Or

Q15-Prove that one of every three consecutive positive integers is divisible by 3.

**YEAR 2022**

**SECTION A (2 MARKS )**

Q-(a) Find the value of \(m\) for which the quadratic equation \((m 1) x^2 + 2 (m 1) x + 1 = 0\) has two real and equal roots

OR

(a) Find the value of \(m\) for which the quadratic equation \((m 1) x^2 + 2 (m 1) x + 1 = 0\) has two real and equal roots

\[\sqrt{3} x^2 + 10x + 7\sqrt{3} = 0\]

Q - The product of Rehan’s age (in years) 5 years ago and his age 7 years from now, is one more than twice his present age. Find his present age.

Q-(a) Find the nature of the roots of the quadratic equation \(x^2 5x + 9 = 0\).

OR

(b) Write a quadratic equation with roots 3 and 5.

Q- Solve the quadratic equation \(2x^2 5x 1 = 0\) for \(x\).

Q-Solve the quadratic equation: \(2+2\sqrt{2}x-6= 0\) for \(x\).

Q- The values of \(x\) and \(y\) satisfying the two equations \(32x + 33y = 34, 33x+32y= 31\) respectively are:

(a) -1,2
(b) -1,4
(c) 1,-2
(d)-1,-4

Q- A quadratic polynomial, the product and sum of whose zeroes are 5 and 8 respectively is

(a) \(kx^28x+5]\)
(b) \(k x^2+8x+5\)
(c) \(kx^2-5x+8]\)
(d) \(kx^2+5x+8\)

Q- Find the roots of the quadratic equation \(9x^2 62x + 2 = 0\).

Q- a) The product of two consecutive odd positive integers is 255. Find the integers, by formulating a quadratic equation.

OR
(b) Find the value(s) of k for the quadratic equation, \((k + 3)x^2 + kx + 1 = 0\), to have two real and equal roots.

**YEAR 2018**

**SECTION A**

Q1-If \(x = 3\) is one root of the quadratic equation \(x^2 - 2kx - 6 = 0\), then find the value of \(k\).

Q2-If one root of \(5x^2 + 13x + k = 0\) is the reciprocal of the other root, then find value of \(k\).

**SECTION-D**

Q-A motor boat whose speed is 18 km/hr in still water takes 1hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

**OR**

A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed?

Q-Obtain all zeroes of \(3x^4 - 15x^3 + 13x^2 + 25x - 30\), if two of its zeroes are \(\sqrt{5}/3\) and \(-\sqrt{5}/3\).

Q-A faster train takes one hour less than a slower train for a journey of 200 km. If the speed of slower train is 10 km/hr less than that of faster train, find the speeds of two trains.

**OR**

Solve for \(x\):

\[
\frac{1}{a + b + x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}, \text{ a not equal to 0, b not equal to 0, x not equal to 0}
\]
**YEAR-2023**

**SECTION A**

Q-The roots of the equation $x^2 + 3x - 10 = 0$ are: (a) 2,-5 (b) -2,5 (c) 2,5 (d) -2,-5

Q-A quadratic equation whose one root is 2 and the sum of whose roots
(a) $x+4=0$
(b) $x-2=0$
(c) $4x-1=0$
(d) $-4-0$

Q-Which of the following is not a quadratic equation?
(a) $2(x-1)=4x-2x+1$
(b) $2x-x^2x^2+5$
(c) $(\sqrt{2}x+\sqrt{3})+x^2=3x^2-5x$
(d) $(x^2+2x)^2 = x^2+3+4x$

**SECTION B**

Q-(A) Find the sum and product of the roots of the quadratic equation $2x^2-9x+4=0$.

OR

B) Find the discriminant of the quadratic equation $4x^2-5=0$ and hence comment on the nature of roots of the equation

Q- Find the discriminant of the quadratic equation $3x^2 -2x+ =0$ and hence find the nature of its roots. OR

Find the roots of the quadratic equation $x^2-x-2=0$

**SECTION C**

Q-Find the value of 'p' for which one root of the quadratic equation $px^2-14x+8=0$ is 6 times the other.

**YEAR-2021**

**SECTION A**

Q-Write the quadratic equation in x whose roots are 2 and -5

**SECTION B**

Q-(a) One root of the quadratic equation $2x^2- 8x- k = 0$ is $5/2$. Find the value of k. Also, find the other root.

OR

(c) Using quadratic formula, solve the following equation for x :

$$abx^2 + (b^2- ac)x -bc = 0$$

**ARITHMETIC PROGRESSION**
SECTION A (1 marks).
1. If in an A.P., $a = 15$, $d = -3$ and $an = 0$, then find the value of $n$.
2. Find the 21st term of the A.P. $2, 1, -1, 1/2, -3, -1 1/2$
3. How many 2-digit numbers are divisible by 3?
4. Write the common difference of the A.P. $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, ..$
5. Which term of the A.P. $-4, -1, 2, ...$ is 101?
6. Find the number of terms in the A.P. $18, 15 1/2, 13, ...,-47$.
7. Find the common difference of the Arithmetic Progression (A.P.) $1/a, 3-a/3a, 3-2a/a,...(a \neq 0)$
8. Find the sum of first 10 multiples of 6.
9. Find the sum of the first 10 multiples of 3.

SECTION B (2 marks)
1. Which term of AP 3, 15, 27, 39,... will be 120 more than it's 21st term?
2. If $S_n$, the sum of first $n$ terms of an AP is by $S_n=3n^2-4n$, find the $n$th term.
3. If $S_n$, the sum of the first $n$ terms of an A.P. is given by $S_n = 2n^2 + n$, then find its term.
4. If the 17th term of an A.P. exceeds its 10th term by 7, find the common difference.
5. How many 2-digit numbers are divisible by 7?
6. If the sum of first $n$ terms of an AP is $n^2$, then find its 10th term.
7. How many multiples of 4 lie between 10 and 205?
8. Determine the A.P. whose third term is 16 and 7th term exceeds the 5th term by 12.

SECTION D (4 marks)
1. Which term of the Arithmetic Progression $-7, -12, -17, -22, ...$ will be $-82$? Is $-100$ any term of the A.P.? Give reason for your answer.
2. How many terms of the Arithmetic Progression $45, 39, 33, ...$ must be taken so that their sum is 180?
   Explain the double answer.
3. The first term of an AP is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of A.P.
4. In an A.P., the first term is $-4$, the last term is 29 and the sum of all its terms is 150. Find its common difference.
5. If the sum of the first $p$ terms of an A.P. is $q$ and the sum of the first $q$ terms is $p$; then show that the sum of the first $(p + q)$ terms is $\{ - (p + q)\}$.
6. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first $n$ terms.
7. In an A.P., the $n$th term is $1/m$ and the $m$th term is $1/n$. Find (i) $(mn)$th term, (ii) sum of first $(mn)$ terms.
8. If $m$ times the $m$th term of an Arithmetic Progression is equal to $n$ times its $n$th term and $m \neq n$, show that the $(m + n)$th term of the A.P. is zero.
9. The sum of the first three numbers in an Arithmetic Progression is 18. If the product of the first and third number is 5 times the common difference, find the numbers.
Class X 2020 AIRTHERMATIC PROGRESSION

Section A (1 MARKS)

1. The common difference of the A.P. \(\frac{1}{p}, \frac{1}{p}, \frac{1}{p} \ldots \) is
   (a) 1 (b) \(\frac{1}{p}\) (c) \(-1\) (d) \(-\frac{1}{p}\)

2. The \(n\)th term of the A.P. \(a, 3a, 5a, \ldots\) is
   (a) \(na\) (b) \((2n - 1)a\) (c) \((2n + 1)a\) (d) \(2na\)

3. The value of \(x\) for which \(2x, (x+10)\) and \((3x+2)\) are the three consecutive terms of A.P, is
   a) 6   b) -6   c) 18   d) -18

4. The first term of an A.P. is \(p\), and the common difference is \(q\), then its 10th term is
   a) \(q + 9p\) b) \(-p - 9q\) c) \(p + 9q\) d) \(2p + 9q\)

5. The common difference of an A.P, whose \(n\)th term is \(an = (3n + 7)\), is
   (a) 3 (b) 7 (c) 10 (d) 6

6. The value of \(p\) for which \((2p + 1), 10\) and \((5p + 5)\) are three consecutive terms of an A.P is
   (a) \(-1\) (b) \(-2\) (c) \(1\) (d) \(2\)

7. The number of terms of an A.P 5, 9, 13, .... 185 is
   (a) 31 (b) 51 (c) 41 (d) 40

8. The first term of an A.P. is 5 and the last term is 45. If the sum of all the terms is 400, the number of terms is
   (A) 20 (B) 8 (C) 10 (D) 16

9. The 9th term of the A.P. \(-15, -11, -7, \ldots\), 49 is
   (A) 32 (B) 0 (C) 17 (D) 13

10. Which of the following is not an A.P. ?
    (A) \(-1, 2, 0.8, 2.8, \ldots\)
    (B) \(3, 3 + 2*2, 3 + 3*2, \ldots\)
    (C) \(4/3, 7/3, 9/3, 12/3\) ....
    (D) \(-1/5, -2/5, -3/5\) ....

11. Find the sum of the first 100 natural numbers.

SECTION B (2 MARKS)

12. Find the 11th term from the last term (towards the first term) of the A.P 12, 8, 4, ..., \(-84\)
13. Find the sum of first 20 terms of the following AP. 1, 4, 7, 10, ….

14. The sum of first 7 terms of AP is 63 and that of its next 7 term is 161. Find the AP

SECTION C (3 MARKS)

15. Show that the sum of all terms of an A.P. whose first term is a, the second term is b and the last term is c is equal to \((a + c)(b + c - 2a)2(b - a)\)

16. Find a, b and c if it is given that the numbers a, 7, b, 23, c are in AP.

17. If \(m\) times the \(m\)th term of an AP is equal to \(n\) times its \(n\)th term, show that the \((m + n)\)th term of the AP is zero.

18. The sum of the first 30 terms of an A.P. is 1920. If the fourth term is 18, find its 11th term.

19. For an A.P., it is given that the first term \((a) = 5\), common difference \((d) = 3\), and the \(n\)th term \((an) = 50\). Find \(n\) and sum of first \(n\) terms \((Sn)\) of the A.P.

20. \(1 + 4 + 7 + 10 + ... + x = 287\).

SECTION D (4 MARKS)

21. The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7:15. Find the numbers.

22. \(1+4+7+10+...+x=287\)

23. If the four times the fourth term is equal to 18 times \(18^{th}\) term, then find \(22^{nd}\) term.

CLASS X 2022 AIRTHEROMATIC PROGRESSION

SECTION A (2 MARKS)

1. Find the sum of first 30 terms of AP: 30, 24, 18, ….
2. In an AP if \(Sn = n(4n + 1)\), then find the AP.
3. Which term of AP, -11/2, -3, -1/2, …, is 49/2?
4. Find a and b so that the numbers a, 7, b, 23 are in AP
5. Find the sum of first 20 terms of AP whose nth term is given as \(a^n=5-2n\)
6. Find the sum of first 20 term of an AP in which \(d=5\) and \(a20=125\)
7. Which value of \(n\) are the nth terms of the APs: 9, 7, 5, …, and 15, 12, 9, … the same?

8. How many natural numbers are there between 1-1000. Which are divisible by 5 but not by 2

SECTION B (3 MARKS)

9. If the last term of an AP of 30 terms is 119 and the 8th term from the end (towards the first term) is 91, then find the common difference of the A.P. Hence, find the sum of all the terms of the A.P.

SECTION C (4 MARKS)

10. In Mathematics, relations can be expressed in various ways. The matchstick patterns are based on linear relations. Different strategies can be used to calculate the number of matchsticks used in different figures.
One such pattern is shown below. Observe the pattern and answer the following questions using Arithmetic Progression:

(a) Write the AP for the number of triangles used in the figures. Also, write the nth term of this AP.

(b) Which figure has 61 matchsticks?

**CLASS X 2023 AIRTHEMATIC PROGRESSION**

**SECTION A**

1. If \( p-1, p + 1 \) and \( 2p + 3 \) are in A.P., then the value of \( p \) is
   (A) -2    (B) 0    (C) 4    (D) 2

2. The next term of the A.P.: \( ^6, ^{24}, ^{54} \) is:
   (a) \( ^{60} \) (b) \( ^{96} \) (c) \( ^{72} \) (d) \( ^{216} \)

3. Assertion (A): \( a, b, c \) are in A.P. if and only if \( 2b = a + c \). Reason (R): The sum of first \( n \) odd natural numbers is \( n^2 \). 1 (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
   (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
   (c) Assertion (A) is true but Reason (R) is false.
   (d) Assertion (A) is false but Reason (R) is true.

4. The common difference of the A.P. whose nth term is given by \( a_n = 3n + 7 \), is:
   (a) 7 (b) 3 (c) 3n (d) 1

5. The 11th term from the end of the A.P.: 10, 7, 4, ....... 62 is:
   (a) 25 (b) 16 (c) 32 (d) 0

6. Assertion (A): The number \( 5n \) cannot end with the digit 0, where \( n \) is a natural number.
   Reason (R): Prime factorisation of 5 has only two factors, 1 and 5.
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true

SECTION C

7. If pth term of an A.P. is q and qth term is p, then prove that its nth term is \((p + q - n)\).

8. How many terms are there in an A.P. whose first and fifth terms are –14 and 2, respectively and the last term is 62.

9. Which term of the A.P. : 65, 61, 57, 53, ............... is the first negative term?

SECTION D

10. The ratio of the 11th term to 17th term of an A.P. is 3 : 4. Find the ratio of 5th term to 21st term of the same A.P. Also, find the ratio of the sum of first 5 terms to that of first 21 terms.

11. 250 logs are stacked in the following manner:

22 logs in the bottom row, 21 in the next row, 20 in the row next to it and so on (as shown by an example). In how many rows, are the 250 logs placed and how many logs are there in the top row?

12. How many terms of the arithmetic progression 45, 39, 33, ........ must be taken so that their sum is 180? Explain the double answer.

CLASS X 2021 AIRTHERMATIC PROGRESSION

SECTION A (1 MARKS)

1. If \(\frac{3}{5}, a, 4\) are three consecutive terms of an A.P., then find the value of \(a\).

2. In an A.P., if the common difference \(d = 3\) and the eleventh term \(a_{11} = 15\), then find the first term

3. In an AP, if the common difference \((d) = -4\) and the seventh term \((a_{7})\) is 4, then find the first term

SECTION B (2 MARKS)

4. Find the sum of first 8 multiples of 3

SECTION D (4 MARKS)
5. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7:15. Find the numbers.

**Triangles**

**1 mark questions**

1- If \( \Delta ABC \sim \Delta RPQ \), \( AB = 3 \) cm, \( BC = 5 \) cm, \( AC = 6 \) cm, \( RP = 6 \) cm and \( PQ = 10 \) cm, then find \( QR \).

2- In \( \Delta DEW \), \( AB \parallel EW \). If \( AD = 4 \) cm, \( DE = 12 \) cm and \( DW = 24 \) cm, then find the value of \( DB \).

3- In the given figure, if \( DE \parallel BC \), \( AE = 8 \) cm, \( EC = 2 \) cm and \( BC = 6 \) cm, then find \( DE \). (2014)

4- In the given figure, \( XY \parallel QR \), \( PQ/XQ=73 \) and \( PR = 6.3 \) cm, find \( YR \). (2017OD)

**2 mark questions**

1- In the figure, \( EF \parallel AC \), \( BC = 10 \) cm, \( AB = 13 \) cm and \( EC = 2 \) cm, find \( AF \). (2014)

2- \( X \) and \( Y \) are points on the sides \( AB \) and \( AC \) respectively of a triangle \( ABC \) such that \( AX/AB=14 \), \( AY = 2 \) cm and \( YC = 6 \) cm. Find whether \( XY \parallel BC \) or not.
3- In the given figure, \( \angle A = 90^\circ \), \( AD \perp BC \). If \( BD = 2 \text{ cm} \) and \( CD = 8 \text{ cm} \), find \( AD \).

(2012; 2017D)

4- In the figure \( ABC \) and \( DBC \) are two right triangles. Prove that \( AP \times PC = BP \times PD \).

5- In the given figure, \( QA \perp AB \) and \( PB \perp AB \). If \( AO = 20 \text{ cm} \), \( BO = 12 \text{ cm} \), \( PB = 18 \text{ cm} \), find \( AQ \)

6- In the figure, \( ABCD \) is a parallelogram and \( E \) divides \( BC \) in the ratio 1:3. \( DB \) and \( AE \) intersect at \( F \). Show that \( DF = 4FB \) and \( AF = 4FE \)

3 mark questions

1. \( ABCD \) is a parallelogram. Point \( P \) divides \( AB \) in the ratio 2:3 and point \( Q \) divides \( DC \) in ratio 4:1. Prove that \( OC \) is half of \( OA \).

2- State and prove basic proportionality theorem.

3- \( ABC \) is a triangle in which \( DE \parallel BC \). If \( AD = x \), \( BD = x-2 \), \( AE = x+2 \) and \( EC = x-1 \), then find the value of \( x \).
4- In the given figure, CD || LA and DE || AC. Find the length of CL if BE = 4 cm and EC = 2 cm.

5- If a line segment intersects sides AB and AC of a \( \triangle ABC \) at D and E respectively and is parallel to BC, prove that \( \frac{AD}{AB} = \frac{AE}{AC} \).

6- A vertical pole of length 8 m casts a shadow 6 cm long on the ground and at the same time a tower casts a shadow 30 m long. Find the height of tower.

7- In the given figure, the line segment XY is parallel to the side AC of \( \triangle ABC \) and it divides the triangle into two parts of equal areas. Find the ratio \( \frac{AX}{AB} \).

8- In the given figure, AD \( \perp \) BC and BD = 13CD. Prove that \( 2AC^2 = 2AB^2 + BC^2 \).

4/5 mark questions

1- Let \( \triangle ABC \) be a triangle and D and E be two points on side AB such that AD = BE. If DP \( \parallel \) BC and EQ \( \parallel \) AC, then prove that PQ \( \parallel \) AB.
2- In the figure, \( \angle BED = \angle BDE \) & E divides BC in the ratio 2 : 1. Prove that \( AF \times BE = 2 \ AD \times CF \).

3- In the given figure, \( AD = 3 \) cm, \( AE = 5 \) cm, \( BD = 4 \) cm, \( CE = 4 \) cm, \( CF = 2 \) cm, \( BF = 2.5 \) cm, then find the pair of parallel lines and hence their lengths.

4- In the figure, PQR and QST are two right triangles, right angled at R and T respectively. Prove that \( QR \times QS = QP \times QT \). (2014)

5- In Figure, \( AB \perp BC \), \( FG \perp BC \) and \( DE \perp AC \). Prove that \( \triangle ADE \sim \triangle GCF \). (2016 OD)

6- In the figure, if \( DE || OB \) and \( EF || BC \), then prove that \( DF || OC \). (2014)
7- In the given figure, CD || LA and DE || AC. Find the length of CL if BE = 4 cm and EC = 2 cm. (2012)

8- In given figure, EB ⊥ AC, BG ⊥ AE and CF ⊥ AE (2015)
Prove that:
(a) ∆ABG ~ ∆DCB
(b) BC/BD = BE/BA

Assertion-Reason Based Questions

1- **Assertion (A):** If ∆ABC and ∆PQR are congruent triangles, then they are also similar triangles.
**Reason (R):** All congruent triangles are similar but the similar triangles need not be congruent.

2- **Assertion (A):** In the given figure, PA || QB || RC || SD.
**Reason (R):** If three or more line segments are perpendiculars to one line, then they are parallel to each other.

3- **Assertion (A):** In the ∆ABC, AB = 24 cm, BC = 10 cm and AC = 26 cm, then ∆ABC is a right angle triangle.
**Reason (R):** If in two triangles, their corresponding angles are equal, then the triangles are similar.
4- **Assertion (A):** If two sides of a right angle are 7 cm and 8 cm, then its third side will be 9 cm.

**Reason (R):** In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

**Case Based Questions**

**CASE STUDY 1:**

Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay’s house is 20m when Vijay’s house casts a shadow 10m long on the ground. At the same time, the tower casts a shadow 50m long on the ground and the house of Ajay casts 20m shadow on the ground.

1. What is the height of the tower?
   
   a) 20m  
   
   b) 50m  
   
   c) 100m  
   
   d) 200m

2. What will be the length of the shadow of the tower when Vijay’s house casts a shadow of 12m?

   a) 75m
3. What is the height of Ajay’s house?
   a) 30m
   b) 40m
   c) 50m
   d) 20m

4. When the tower casts a shadow of 40m, same time what will be the length of the shadow of Ajay’s house?
   a) 16m
   b) 32m
   c) 20m
   d) 8m

5. When the tower casts a shadow of 40m, same time what will be the length of the shadow of Vijay’s house?
   a) 15m
   b) 32m
   c) 16m
   d) 8m

**CASE STUDY 2:**

A scale drawing of an object is the same shape as the object but a different size. The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio. The ratio of two corresponding sides in similar figures is called the scale factor.
If one shape can become another using revising, then the shapes are similar. Hence, two shapes are similar when one can become the other after a resize, flip, slide or turn. In the photograph below showing the side view of a train engine.

Scale factor is 1:200

This means that a length of 1 cm on the photograph above corresponds to a length of 200cm or 2 m, of the actual engine. The scale can also be written as the ratio of two lengths.

1. If the length of the model is 11cm, then the overall length of the engine in the photograph above, including the couplings (mechanism used to connect) is:
   a) 22cm
   b) 220cm
   c) 220m
   d) 22m

2. What will affect the similarity of any two polygons?
   a) They are flipped horizontally
   b) They are dilated by a scale factor
   c) They are translated down
d) They are not the mirror image of one another.

3. What is the actual width of the door if the width of the door in photograph is 0.35cm?

a) 0.7m  
b) 0.7cm  
c) 0.07cm  
d) 0.07m

4. If two similar triangles have a scale factor 5:3 which statement regarding the two triangles is true?

a) The ratio of their perimeters is 15:1  
b) Their altitudes have a ratio 25:15  
c) Their medians have a ratio 10:4  
d) Their angle bisectors have a ratio 11:5

5. The length of AB in the given figure:

a) 8cm  
b) 6cm  
c) 4cm
d) 10cm

# COORDINATE GEOMETRY

## 2023

|   | The ratio in which the x-axis divides the line segment joining the points (2, 3) and (6, 7) is:  
|   | (a) 1 : 3 (b) 3 : 7 (c) 7 : 3 (d) 1 : 2  
| 2 | Show that the points (2, 3), (8, 3) and (6, 7) are the vertices of a right-angled triangle.  
| 3 | If Q(0, 1) is equidistant from P(5, 3) and R(x, 6), find the values of x.  
| 4 | The coordinates of the vertex A of a rectangle ABCD whose three vertices are given as B(0, 0), C(3, 0) and D(0, 4) are:  
|   | (a) (4, 0) (b) (0, 3) (c) (3, 4) (d) (4, 3)  
| 5 | The distance of the point (–6, 8) from origin is:  
|   | (a) 6 (b) –6 (c) 8 (d) 10  
| 6 | The distance of the point (–1, 7) from x-axis is:  
|   | (a) –1 (b) 7 (c) 6 (d) √50  
| 7 | The points (–4, 0), (4, 0) and (0, 3) are the vertices of a:  
|   | (a) right triangle (b) isosceles triangle (c) equilateral triangle (d) scalene triangle  
| 8 | The distance between the points P(–11/3, 5) and Q(–2/3, 5) is:  
|   | (a) 6 units (b) 4 units (c) 2 units (d) 3 units  
| 9 | The line segment joining the points A(4, 5) and B(4, 5) is divided by the point P such that AP : AB = 2 : 5. Find the coordinates of P.  
| 10 | Point P(x, y) is equidistant from points A(5, 1) and B(1, 5). Prove that x = y.  
|   | Find the ratio in which y-axis divides the line segment joining the points (5, 6) and (1, 4).  
|   | A line intersects y-axis and x-axis at point P and Q, respectively. If R(2, 5) is the mid-point of line segment PQ, then find the coordinates of P and Q.  
| 13 | In what ratio, does x-axis divide the line segment joining the points A(3, 6) and B(–12, –3)?  
|   | (A) 1 : 2 (B) 1 : 4  
|   | (C) 4 : 1 (D) 2 : 1  
| 14 | The distance between the points (0, 2√5) and (–2√5, 0) is  
|   | (A) 2√10 units (B) 4√10 units  
|   | (C) 2√20 units (D) 0
Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.
Based on the above information, answer the following questions:
(i) Taking O as origin, coordinates of P are (−200, 0) and of Q are (200, 0). PQRS being a square, what are the coordinates of R and S?
(ii) (a) What is the area of square PQRS?
OR
(b) What is the length of diagonal PR in square PQRS?
(iii) If S divides CA in the ratio K:1, what is the value of K, where point A is (200, 800)?

2022

1 Check whether the points P(5, 2), Q(6, 4) and R(7, 2) are the vertices of an isosceles triangle PQR.

2 Find the ratio in which P(4, 5) divides the join of A(2, 3) and B(7, 8).

3 Students of a school are standing in rows and columns in their school playground to celebrate their annual sports day. A, B, C and D are the positions of four students as shown in the figure.

Based on the above, answer the following questions:
(i) The figure formed by the four points A, B, C and D is a 1 (A) square
(B) parallelogram
(C) rhombus
(D) quadrilateral

(ii) If the sports teacher is sitting at the origin, then which of the four students is closest to him?
(A) A
(B) B
(C) C
(D) D

(iii) The distance between A and C is
(A) √37 units
(B) √35 units
(C) 6 units
(D) 5 units

(iv) The coordinates of the mid-point of line segment AC are
(A) 5/2, 11
(B) 5/2, 11/2
(C) 5, 11/2
(D) (5, 11)

(v) If a point P divides the line segment AD in the ratio 1 : 2, then coordinates of Pare
(A) 8/3, 8/3
(B) 10/3, 13/3
(C) 13/3, 10/3
(D) 16/3, 11/3

2020

1. The point P on x-axis equidistant from the points A(−1, 0) and B(5, 0) is
(a) (2, 0) (b) (0, 2) (c) (3, 0) (d) (2, 2)

2. The co-ordinates of the point which is reflection of point (−3, 5) in x-axis are
(a) (3, 5) (b) (3, −5) (c) (−3, −5) (d) (−3, 5)

3. If the point P (6, 2) divides the line segment joining A(6, 5) and B(4, y) in the ratio 3 : 1, then the value of y is
(a) 4 (b) 3 (c) 2 (d) 1

4. If (a, b) is the mid-point of the line segment joining the points A(10, −6) and B(k, 4) and a − 2b = 18, the value of k is
(a) 30 (b) 22 (c) 4 (d) 40

5. The distance between the points (m, −n) and (−m, n) is
(A) √m² + n²
(B) m + n
(C) 2√m² + n²
(D) √2m² + 2n²

2019
1. Find the coordinates of a point A, where AB is a diameter of the circle with centre (−2, 2) and B is the point with coordinates (3, 4).

2. The line segment joining the points A(2, 1) and B(5, −8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by 2x − y + k = 0, find the value of k.

3. Points A(3, 1), B(5, 1), C(a, b) and D(4, 3) are vertices of a parallelogram ABCD. Find the values of a and b.

4. Points P and Q trisect the line segment joining the points A(−2, 0) and B(0, 8) such that P is nearer to A. Find the coordinates of points P and Q.

5. Point P divides the line segment joining the points A(2, 1) and B(5, −8) such that \( \frac{AP}{AB} = \frac{1}{3} \). If P lies on the line 2x − y + k = 0, find the value of k.

---

INTRODUCTION TO TRIGONOMETRY

2023

1. If \( 4 \cot^2 45 \sec^2 60 + \sin^2 60 + p = \frac{3}{4} \), then find the value of p.

2. If \( \cos A + \cos^2 A = 1 \), then find the value of \( \sin^2 A + \sin^4 A \).

3. \[
\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \csc \theta
\]

PROVE THAT:

4. Prove that:

\[
\left( \frac{1}{\cos \theta} - \cos \theta \right) \left( \frac{1}{\sin \theta} - \sin \theta \right) = \frac{1}{\tan \theta + \cot \theta}.
\]

5. \[
\frac{\cos^2 \theta}{1 - \tan \theta} + \frac{\sin^3 \theta}{\sin \theta - \cos \theta} = 1 + \sin \theta \cos \theta
\]

6. \( \sec \theta \) when expressed in terms of \( \cot \theta \), is equal to:

(a) \( \frac{1 + \cot^2 \theta}{\cot \theta} \)  
(b) \( \sqrt{1 + \cot^2 \theta} \)

(c) \( \frac{\sqrt{1 + \cot^2 \theta}}{\cot \theta} \)  
(d) \( \sqrt{1 - \cot^2 \theta} \)

7. If A and B are acute angles such that \( \sin (A − B) = 0 \) and \( 2 \cos (A + B) − 1 = 0 \), then find angles A and B.
8. Evaluate \[ \frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ} \]

9. Prove that \[ \frac{\sin A - 2 \sin^3 A}{2 \cos^3 A - \cos A} = \tan A \]

10. Prove that \( \sec A (1 - \sin A) (\sec A + \tan A) = 1 \).

11. Which of the following is true for all values of \( \theta \) \( (0^\circ \leq \theta \leq 90^\circ) \)?
   (a) \( \cos^2 \theta - \sin^2 \theta = 1 \)  
   (b) \( \cosec^2 \theta - \sec^2 \theta = 1 \)  
   (c) \( \sec^2 \theta - \tan^2 \theta = 1 \)  
   (d) \( \cot^2 \theta - \tan^2 \theta = 1 \)

12. (A) Evaluate: \[ \frac{5}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cot^2 45^\circ + 2 \sin^2 90^\circ \]

   **OR**

   (B) If \( \theta \) is an acute angle and \( \sin \theta = \cos \theta \), find the value of \( \tan^2 \theta + \cot^2 \theta - 2 \).

13. \((\sec^2 \theta - 1) (\cosec^2 \theta - 1)\) is equal to:
   (a) \(-1\)  
   (b) \(1\)  
   (c) \(0\)  
   (d) \(2\)

14. If \( \tan \theta = \frac{5}{12} \), then the value of \( \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} \) is:
   (a) \(-\frac{17}{7}\)  
   (b) \(\frac{17}{7}\)  
   (c) \(\frac{17}{13}\)  
   (d) \(-\frac{7}{13}\)

   \( \left( \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} \right) \) is equal to:
   (a) \(\sin 60^\circ\)  
   (b) \(\cos 60^\circ\)  
   (c) \(\tan 60^\circ\)  
   (d) \(\sin 30^\circ\)

15. (a) If \( a \cos \theta + b \sin \theta = m \) and \( a \sin \theta - b \cos \theta = n \), then prove that \( a^2 + b^2 = m^2 + n^2 \).

   **OR**

   (b) Prove that:
   \[ \frac{\sqrt{\sec A - 1}}{\sec A + 1} + \frac{\sqrt{\sec A + 1}}{\sec A - 1} = 2 \cosec A \]
\[
\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \csc \theta
\]

If \( \sec \theta - \tan \theta = \frac{1}{3} \), then the value of \( (\sec \theta + \tan \theta) \) is:

(a) \( \frac{4}{3} \)  
(b) \( \frac{2}{3} \)

(c) \( \frac{1}{3} \)  
(d) 3

\[
\frac{\tan A}{1 + \sec A} - \frac{\tan A}{1 - \sec A} = 2 \csc A
\]

\[
\frac{\cos^2 \theta}{\sin^2 \theta} - \frac{1}{\sin^2 \theta}, \text{ in simplified form, is :}
\]

(A) \( \tan^2 \theta \)  
(B) \( \sec^2 \theta \)

(C) 1  
(D) -1

If \( \theta \) is an acute angle of a right angled triangle, then which of the following equation is not true?

(A) \( \sin \theta \cot \theta = \cos \theta \)  
(B) \( \cos \theta \tan \theta = \sin \theta \)

(C) \( \csc^2 \theta - \cot^2 \theta = 1 \)  
(D) \( \tan^2 \theta - \sec^2 \theta = 1 \)

**Statement A (Assertion)**: For \( 0 < \theta \leq 90^\circ \), \( \csc \theta - \cot \theta \) and \( \csc \theta + \cot \theta \) are reciprocal of each other.

**Statement R (Reason)**: \( \csc^2 \theta - \cot^2 \theta = 1 \)

Prove that \( \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A} \).

If \( 2 \tan A = 3 \), then the value of \( \frac{4 \sin A + 3 \cos A}{4 \sin A - 3 \cos A} \) is:

(A) \( \frac{7}{\sqrt{13}} \)  
(B) \( \frac{1}{\sqrt{13}} \)

(C) 3  
(D) does not exist
\[ \frac{3}{4} \tan^2 30^\circ - \sec^2 45^\circ + \sin^2 60^\circ \] is equal to

(A) \(-1\) \hspace{1cm} (B) \(\frac{5}{6}\)
(C) \(-\frac{3}{2}\) \hspace{1cm} (D) \(\frac{1}{6}\)

Prove that: \(\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}\)

(a) यदि \(\sin \theta + \cos \theta = \sqrt{3}\) है, तो \(\sin \theta \cdot \cos \theta\) का मान ज्ञात कीजिए।

अथवा

(b) यदि \(\sin \alpha = \frac{1}{\sqrt{2}}\) और \(\cot \beta = \sqrt{3}\) है, तो \(\csc \alpha + \csc \beta\) का मान ज्ञात कीजिए।

(a) If \(\sin \theta + \sin^2 \theta = 1\), then prove that \(\cos^2 \theta + \cos^4 \theta = 1\).

OR

(b) If \(\tan \theta = \frac{1}{\sqrt{7}}\), then show that \(\frac{\csc^2 \theta - \sec^2 \theta}{\csc^2 \theta + \sec^2 \theta} = \frac{3}{4}\).

Prove that: \(2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = \frac{\cos^2 \theta}{\sin^2 \theta} - \frac{1}{\sin^2 \theta}\), in simplified form, is:

(A) \(\tan^2 \theta\) \hspace{1cm} (B) \(\sec^2 \theta\)
(C) \(1\) \hspace{1cm} (D) \(-1\)

\((\cos^4 A - \sin^4 A)\) on simplification, gives

(A) \(2 \sin^2 A - 1\) \hspace{1cm} (B) \(2 \sin^2 A + 1\)
(C) \(2 \cos^2 A + 1\) \hspace{1cm} (D) \(2 \cos^2 A - 1\)

If \(\sin \theta + \cos \theta = p\) and \(\sec \theta + \cosec \theta = q\), then prove that \(q(p^2 - 1) = 2\)

If \(\theta\) is an acute angle of a right angled triangle, then which of the following equation is not true?

(A) \(\sin \theta \cot \theta = \cos \theta\) \hspace{1cm} (B) \(\cos \theta \tan \theta = \sin \theta\)
(C) \(\cosec^2 \theta - \cot^2 \theta = 1\) \hspace{1cm} (D) \(\tan^2 \theta - \sec^2 \theta = 1\)
If \( \tan \theta = \frac{x}{y} \), then cos \( \theta \) is equal to

| (A) | \( \frac{x}{\sqrt{x^2 + y^2}} \) | (B) | \( \frac{y}{\sqrt{x^2 + y^2}} \) |
| (C) | \( \frac{x}{\sqrt{x^2 - y^2}} \) | (D) | \( \frac{y}{\sqrt{x^2 - y^2}} \) |

2021

1 (a) Prove:

\[
\frac{1}{(\cot A) (\sec A) - \cot A} - \cosec A = \cosec A - \frac{1}{(\cot A) (\sec A) + \cot A}
\]

OR

(b) Prove:

\[
\sin^6 A + 3 \sin^2 A \cos^2 A = 1 - \cos^6 A
\]

2 If \( \tan \theta + \cot \theta = \sqrt{3} / 3 \), then find the value of \( \tan^2 \theta + \cot^2 \theta \).

2020

1 Prove that \( 1 + \frac{\cot^2 \alpha}{1 + \cosec \alpha} = \cosec \alpha \)

OR

Show that \( \tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta \)

2 If \( \sin \theta + \cos \theta = \sqrt{2} \), prove that \( \tan \theta + \cot \theta = 2 \).

3 Prove that \( 1 + \frac{\cot^2 \alpha}{1 + \cosec \alpha} = \cosec \alpha \)

OR

Show that \( \tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta \)
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| 4 | Prove that:  \[
\frac{1 + \sin A}{\sqrt{1 - \sin A}} = \sec A + \tan A
\] |

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**APPLICATION TO TRIGONOMETRY**

**2023**

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>The length of the shadow of a tower on the plane ground is ( \sqrt{3} ) times the height of the tower. Find the angle of elevation of the sun.</td>
</tr>
<tr>
<td>2</td>
<td>The angle of elevation of the top of a tower from a point on the ground which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.</td>
</tr>
<tr>
<td>3</td>
<td>The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also the height of the other tower.</td>
</tr>
<tr>
<td>4</td>
<td>From the top of a tower 100 m high, a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the two cars. (Use ( \sqrt{3} = 1.73 ))</td>
</tr>
</tbody>
</table>
5. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30 and 60. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. (Use \( \sqrt{3} = 1.73 \))

6. From a point on the ground, the angle of elevation of the bottom and top of a transmission tower fixed at the top of 30 m high building are 30 and 60, respectively. Find the height of the transmission tower. (Use \( \sqrt{3} = 1.73 \))

7. If a pole 6 m high casts a shadow 2.3 m long on the ground, then sun’s elevation is:
   (a) 60°  (b) 45°  (c) 30°  (d) 90°

8. A straight highway leads to the foot of a tower. A man standing on the top of the 75 m high tower observes two cars at angles of depression of 30° and 60°, which are approaching the foot of the tower. If one car is exactly behind the other on the same side of the tower, find the distance between the two cars. (Use \( \sqrt{3} = 1.73 \))

9. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.

10. One observer estimates the angle of elevation to the basket of a hot air balloon to be 60°, while another observer 100 m away estimates the angle of elevation to be 30°. Find:
    (a) The height of the basket from the ground.
    (b) The distance of the basket from the first observer’s eye.
    (c) The horizontal distance of the second observer from the basket.

11. The angle of elevation of the top of a vertical tower from a point P on the ground is 60°. From another point Q, 10 m vertically above the first point P, its angle of elevation is 30°. Find:
    (a) The height of the tower.
    (b) The distance of the point P from the foot of the tower.
    (c) The distance of the point P from the top of the tower.

12. Two pillars are standing on either side of a 80 m wide road. Height of one pillar is 20 m more than the height of the other pillar. From a point on the road between the pillars, the angle of elevation of the higher pillar is 60°, whereas that of the other pillar is 30°. Find the position of the point between the pillars and the height of each pillar. (Use \( \sqrt{3} = 1.73 \))

13. Find the length of the shadow on the ground of a pole of height 18 m when angle of elevation \( \theta \) of the sun is such that \( \tan \theta = \frac{6}{7} \).

Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure.

On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point Q.

Distance between the base of the tower and point Q is 36 cm. From point Q, the angle of elevation of the top of Section B is 30° and the angle of elevation of the top of Section A is 45°.
Based on the above information, answer the following questions:

(i) Find the length of the wire from the point O to the top of Section B.
(ii) Find the height of the Section A from the base of the tower.
(iii) Find the distance AB.

The angle of elevation of the top of a tower 24 m high from the foot of another tower in the same plane is 60°. The angle of elevation of the top of second tower from the foot of the first tower is 30°. Find the distance between two towers and the height of the other tower. Also, find the length of the wire attached to the tops of both the towers.

A spherical balloon of radius r subtends an angle of 60° at the eye of an observer. If the angle of elevation of its centre is 45° from the same point, then prove that height of the centre of the balloon is $\sqrt{2}$ times its radius.

A ladder set against a wall at an angle $45^\circ$ to the ground. If the foot of the ladder is pulled away from the wall through a distance of 4 m, it slides a distance of 3 m down the wall making an angle $30^\circ$ with the ground. Find the final height of the top of the ladder from the ground and length of the ladder.

An aeroplane when flying at a height of 3000 m from the ground vertically above another aeroplane at an instant when the angle of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. Also, find the distance of first plane from the point of observation.

(Take $\sqrt{3} = 1.73$)
3. Gadisar Lake is located in the Jaisalmer district of Rajasthan. It was built by the King of Jaisalmer and rebuilt by Gadsi Singh in 14th century. The lake has many Chhatris. One of them is shown below:

Observe the picture. From a point A h m above from water level, the angle of elevation of top of Chhatri (point B) is 45 and angle of depression of its reflection in water (point C) is 60°. If the height of Chhatri above water level is (approximately) 10 m, then
(a) draw a well-labelled figure based on the above information;
(b) find the height (h) of the point A above water level.
(Use \( \sqrt{3} = 1.73 \))

4. An aeroplane when flying at a height of 3125 m from the ground passes vertically below another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 30° and 60° respectively. Find the distance between the two planes at that instant.

5. A man standing on the deck of a ship, which is 10 m above the water level, observes that the angle of elevation of the top of a hill is 60° and the angle of depression of the base of the hill is 30°. Find the height of the hill.

6. The shadow of a tower standing on a level ground is found to be 40 m longer when the SUN’S altitude is 30° than when it is 60°. Find the height of the tower.

7. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of Q is 45°. Find the height of the tower PQ and the distance XP. (Use \( \sqrt{3} = 1.732 \))

2021

1. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 18 minutes for the angle of depression to change from 30 to 60°, how soon after this will the car reach the tower?

2. Find the length of the shadow on the ground of a pole of height 18 m when angle of elevation \( \theta \) of the sun is such that \( \tan \theta = \frac{6}{7} \).

2020

1. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

2. If the angle of elevation of a cloud from a point 10 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°, find the height of the cloud from the surface of lake.
### 3
A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag – staff of height \( h \). At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of \( h \).

### 4
A statue 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal. (Use \( \sqrt{3} = 1.73 \))

### 2019

### 1
A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use \( \sqrt{3} = 1.732 \)]

### 2
Amit, standing on a horizontal plane, finds a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, finds the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.

### 3
A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of a 20 m high building, finds the elevation of the same bird to be 45°. The boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl. (Given \( \sqrt{2} = 1.414 \))

### 4
The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of 3600 m, find the speed of the aeroplane.

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### MATHEMATICS

### CIRCLES

#### Very Short Answer Type Questions (VSQs):

1. Two concentric circles of radii \( a \) and \( b \) (\( a > b \)) are given. Find the length of the chord of the larger circle which touches the smaller circle. [CBSE 2019] [1 Mark]
2. In the given figure, TP and TQ are tangents drawn to the circle with centre at O. If \( \angle POQ = 115° \) then \( \angle PTQ \) is [CBSE 2020] [1 Mark]
3. From an external point \( Q \), the length of the tangent to a circle is 5 cm and the distance of \( Q \) from the centre is 8 cm. The radius of the circle is [CBSE 2020] [1 Mark]
4. In the given figure \( AB \) and \( CD \) are common tangents to circles which touch each other at \( D \). If \( AB = 8 \) cm, then find the length of \( CD \). [CBSE 2020] [1 Mark]
5. In the given figure \( PQ \) is a tangent at a point \( C \) to a circle with centre \( O \). If \( AB \) is a diameter and \( \angle CAB = 30° \). Find \( \angle PCA \). [CBSE 2016] [1 Mark]
Short Answer Type Questions (SAQs):

1. Prove that the lengths of tangents drawn from an external point to a circle are equal. [CBSE 2016] [2 Marks]
2. In the given figure, O is the centre of a circle, PQ is a chord and PT is the tangents at P. If \( \angle POQ = 70^\circ \) then find \( \angle TPQ \). [CBSE 2017] [2 Marks]
3. In the given figure, find the length of AP, where O is the centre of circle. [CBSE 2017] [2 Marks]
4. In the given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If OP = 2r, show that \( \angle OTS = \angle OST = 30^\circ \). [CBSE 2016] [2 Marks]
5. In the given figure a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that \( AB + CD = BC + DA \). [CBSE 2016] [2 Marks]

Long Answer Type Questions (LAQs):

1. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. [CBSE 2019] [3 Marks]
2. In the given figure, AB is the diameter of a circle with centre O and AC is its chord such that \( \angle BAC = 30^\circ \). If the tangent drawn at C intersects extended AB at D, then show that BC = BD. [CBSE 2020] [3 Marks]
3. In the given figure, two equal circles, with centres O and O', touch each other at X. OO' produced meets the circle with centre O' at A. AC is tangent to circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of DO'/CO. [CBSE 2016] [4 Marks]
4. TP and TQ are two tangents drawn to a circle with centre O from an external point T. Prove that \( \angle PTQ = 2\angle OPQ \). [CBSE 2017] [4 Marks]
5. A model of a traffic signal on the road has a triangular base ABC with \( \angle A = 90^\circ \) and with a red circular light within it as shown in the figure. If AB = 12 cm and BC =20 cm and R is the in centre of the triangle ABC, find the area used for the red light. [CBSE 2017] [4 Marks]

Areas Related to Circles

Long Answer Type Questions (LAQs):

1. In the given figure, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. (Take \( n = 3.14 \)) [CBSE 2016] [3 Marks]
2. In the given figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where \( \angle AOC = 40^\circ \). (Use \( n = 22/7 \)) [CBSE 2016] [3 Marks]
3. A momento is made as shown in the figure. Its base PBCR is silver plated from the front side. Find the area which is silver plated. (Use \( n = 22/7 \)) [CBSE 2017] [3 Marks]
4. Find the area of the shaded region in figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid point P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD of side 12 cm. (Take \( n = 3.14 \)) [CBSE 2018] [3 Marks]
5. A chord of a circle of radius 14 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment of the circle. (Use \( n = 22/7 \) and \( \sqrt{3} = 1.73 \)) [CBSE 2019] [3 Marks]
6. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle 120°. Find the total area cleaned at each sweep of the blades. (Use \( n = 22/7 \)) [CBSE 2019] [3 Marks]
7. In the given figure, find the area of the shaded region where a circular arc of radius 7 cm has been drawn with vertex O of an equilateral triangle OAB of side 14 cm as centre. (Use \( \pi = \frac{22}{7} \) and \( \sqrt{3} = 1.73 \)) [CBSE 2020] [3 Marks]

8. Calculate the area of the shaded region common between two quadrants of circles of radius 7 cm each (as shown in figure). [CBSE 2020] [3 Marks]

9. In the given figure, is shown a sector OAP of a circle with centre O, containing \( \angle \theta \). AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is \( \pi \tan \theta + \sec \theta + 180 - 1 \). [CBSE 2016] [4 Marks]

10. In a given figure, triangle ABC is a right angled triangle in which \( \angle A \) is 90°. Semicircle are drawn on AB, AC and BC as diameters. Find the area of shaded region. [AI 2017]

**SURFACE AREA AND VOLUME**

1- Savita has a lamp placed at the center of her square yard, each side measuring 20 m. the light of lamp covers a circle of radius 10 m on yard. What area of the yard is not lit by the lamp.

[a] 400 \( \pi \) sq m    [b] 100 \( \pi \) sq m    [c] (40-10 \( \pi \)) sq m    [d] (400-100 \( \pi \)) sq m

2- If two solid hemisphere of same base radius ‘r’ are joined together along their bases, Curved surface area of new solid is .................

[a] 4\( \pi \) \( r^2 \)    [b] 6\( \pi \) \( r^2 \)    [c] 3\( \pi \) \( r^2 \)    [d] 8\( \pi \) \( r^2 \)

3- A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface area of the hemispherical part and the conical part are equal, find the ratio of the radius and the height of the conical part?

4- A solid is in the form of cylinder with hemispherical end. The total height of the solid is 20cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid?

5- A wooden article was made by scooping out a hemisphere from each end of a solid cylinder as shown in figure. If the height of the cylinder is 10 cm and its base radius is 3.5 cm. find the total surface area of the article.
6- A toy is in the form of the cone of radius 3.5 cm mounted on the hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. find the total surface area of toy?

7- In the sum of the radius of the base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq cm. find the volume of the cylinder. (use \( \pi = \frac{22}{7} \)).

8- From a solid cylinder whose height is 8 cm and radius is 6 cm, conical cavity of same height and same base radius is hollowed out, find the total surface area of remaining solid? (Take \( \pi = 3.14 \))

9- A solid iron pole consists of a cylinder of height 220 cm and base diameter is 24 cm which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole given 1 cm\(^3\) of iron has approximately 8 gm mass. (\( \pi = 3.14 \))

10- A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter can have? Find the surface area of the solid?

11- From a solid cylinder whose height is 2.4 cm and a diameter 1.4 cm, a conical cavity of the same height and same diameter hollowed out. Find the total surface area of the remaining solid to the nearest cm\(^2\)?

12- The curved surface area of a right circular cylinder is 176 sq cm and its volume is 1232 cubic cm. Find the height of the cylinder.

13- A- Assertion. If the volume of two spheres are in the ratio of 64:27. The ratio of their surface area is 4:3

B- Reason- if the surface area of two spheres are in the ratio 16:9. Ratio of their volume is 64:27.

[a] both a and r are correct and r is the correct explanation of a
[b] both a and r are true but r is not the correct explanation of a
[c] a is true but r is false
[d] a is false but r is true

14- assertion (a) rampal desided to donate canvas for 10 tents conical in shape with base diameter 14 m and height 24 m to a center for handicapped person’s welfare. The slant height of the conical tents is 25.
Reason- according to assertion, the surface area of 10 tents is 5500 m².

15- a student was asked to make a modal shaped like a cylinder with two cones attached to its ends by using a thin alluminium sheets. The diameter of the modal is 3 cm and its total length is 12 cm. if each cone has a height of 2 cm, find the volume of the air contained in the modal.

16- Atend is in the shape of a cylinder surmounted by a conical top. If the height and the radius of the cylinder parts are 3 m and 14 m respectively and the total height of the tents is 13.5 m. find the area of canvas required for making the tent. If 26m² canvas is used for stitching and wastage also find cost of canvas at the ratio of rs 500 per m².

STATISTICS

1- Find the upper limit of the modal class from the given distribution

<table>
<thead>
<tr>
<th>Height (in cm)</th>
<th>Below 140</th>
<th>Below 145</th>
<th>Below 150</th>
<th>Below 155</th>
<th>Below 160</th>
<th>Below 165</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of girls</td>
<td>4</td>
<td>11</td>
<td>29</td>
<td>40</td>
<td>46</td>
<td>51</td>
</tr>
</tbody>
</table>

2- The length of 40 leaves of a plant are measured correct to nearest millimetre and the data obtained is represented in the following table.

<table>
<thead>
<tr>
<th>Length (in mm)</th>
<th>No. of leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>118-126</td>
<td>3</td>
</tr>
<tr>
<td>127-135</td>
<td>5</td>
</tr>
<tr>
<td>136-144</td>
<td>9</td>
</tr>
<tr>
<td>145-153</td>
<td>12</td>
</tr>
<tr>
<td>154-162</td>
<td>5</td>
</tr>
<tr>
<td>163-171</td>
<td>4</td>
</tr>
<tr>
<td>172-180</td>
<td>2</td>
</tr>
</tbody>
</table>

Find the average length of leaves.

3- The median of the following data is 50. Find the values of p and q, if some of all frequencies is 90. Also find the Mode.

<table>
<thead>
<tr>
<th>Marks obtained</th>
<th>No. of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-30</td>
<td>p</td>
</tr>
<tr>
<td>30-40</td>
<td>15</td>
</tr>
<tr>
<td>40-50</td>
<td>25</td>
</tr>
<tr>
<td>50-60</td>
<td>20</td>
</tr>
<tr>
<td>60-70</td>
<td>q</td>
</tr>
<tr>
<td>70-80</td>
<td>8</td>
</tr>
<tr>
<td>80-90</td>
<td>10</td>
</tr>
</tbody>
</table>

4- The empirical relation between the mode, median and mean of distribution is

[a] mode = 3 median – 2 mean    [b] mean= 3 mean – m2 median
[c] mode= 2 median-3 mean       [d] mode= 2 mean- 3 median

5- The monthly expenditure on milk in 200 families of a housing society is given below
Find the value of x. Find the median and mean expenditure on milk.

6- For the following distribution

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>0-5</th>
<th>5-10</th>
<th>10-15</th>
<th>15-20</th>
<th>20-25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10</td>
<td>15</td>
<td>12</td>
<td>20</td>
<td>9</td>
</tr>
</tbody>
</table>

The sum of lower limits of median class and modal class is


7- For the following distribution

<table>
<thead>
<tr>
<th>Marks below</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of students</td>
<td>3</td>
<td>12</td>
<td>27</td>
<td>57</td>
<td>75</td>
<td>80</td>
</tr>
</tbody>
</table>

The modal class is


8- 250 apples of a box were weighted and the distribution of masses of the apples is given in the following table

<table>
<thead>
<tr>
<th>Masses [in g]</th>
<th>80-100</th>
<th>100-120</th>
<th>120-140</th>
<th>140-160</th>
<th>160-180</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of apples</td>
<td>20</td>
<td>60</td>
<td>70</td>
<td>x</td>
<td>60</td>
</tr>
</tbody>
</table>

[a] Find the value of x and the mean mass of the apples.

[b] Find the modal mass of apples.
9- If mode of the following frequency distribution is 55, find the value of x.

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>0-15</th>
<th>15-30</th>
<th>30-45</th>
<th>45-60</th>
<th>60-75</th>
<th>75-90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10</td>
<td>7</td>
<td>x</td>
<td>15</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

10- The median of the following data is 16. Find the missing frequencies a and b, if the total of the frequencies is 70.

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>0-5</th>
<th>5-10</th>
<th>10-15</th>
<th>15-20</th>
<th>25-30</th>
<th>35-40</th>
<th>40-45</th>
<th>45-50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequencies</td>
<td>12</td>
<td>a</td>
<td>12</td>
<td>15</td>
<td>b</td>
<td>6</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

11- The mode of the following data is 67. Find the missing frequencies x.

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>x</td>
<td>15</td>
<td>12</td>
<td>7</td>
</tr>
</tbody>
</table>

12- Find the mean of the following data using assumed mean method.

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>0-5</th>
<th>5-10</th>
<th>10-15</th>
<th>15-20</th>
<th>20-25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>8</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>12</td>
</tr>
</tbody>
</table>

13- The median of the following data is 525, find the value of x and y if total of frequency is 100.

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-100</td>
<td>2</td>
</tr>
<tr>
<td>100-200</td>
<td>5</td>
</tr>
<tr>
<td>200-300</td>
<td>x</td>
</tr>
<tr>
<td>300-400</td>
<td>12</td>
</tr>
<tr>
<td>400-500</td>
<td>17</td>
</tr>
<tr>
<td>500-600</td>
<td>20</td>
</tr>
<tr>
<td>600-700</td>
<td>y</td>
</tr>
<tr>
<td>700-800</td>
<td>9</td>
</tr>
<tr>
<td>800-900</td>
<td>7</td>
</tr>
<tr>
<td>900-1000</td>
<td>4</td>
</tr>
</tbody>
</table>
14- If the median of the following frequency distribution is 32.5. Find the values of x and y.

<table>
<thead>
<tr>
<th>Class interval</th>
<th>0-10</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>x</td>
<td>5</td>
<td>9</td>
<td>12</td>
<td>y</td>
<td>3</td>
<td>2</td>
<td>40s</td>
</tr>
</tbody>
</table>

**Probability**

Q1. Ginny flipped a fair coin three times and tails came up each time. Ginny wants to flip the coin again. What is the probability of getting heads in the next coin flip?

(a) 0    (b) 0.25   (c) 0.5   (d) 1

Q2. There is a square board of side 2a units circumscribing a red circle. Jayadev is asked to keep a dot on the above said board. The probability that he keeps the dot on outside the circle is

(a) \(\pi/4\)   (b) \((4-\pi)/4\)   (c) \((\pi-4)/4\)   (d) 0.7

Q3. Two cards of heart and 4 cards of spade are missing from a pack of 52 cards. A card is drawn at random from the remaining pack. What is the probability of getting a black cards?

(a) 22/56   (b) 26/52   (c) 24/52   (d) 24/46

Q4. Two dice are thrown together. The probability of getting the difference of numbers on their upper faces equals to 3 is

(a) \(1/9\)   (b) 2/9   (c) 1/6   (d) 1/12

Q5. A card is drawn at random from a well shuffled pack of 52 cards. The probability that the card drawn is not an ace is

(a) 1/13   (b) 9/13   (c) 4/13   (d) 12/13

Q6. Assertion (A): The probability that a leap year has 53 Sundays is \(2/7\).

Reason (R): The probability that a non leap year has 53 Sundays is \(5/7\)

(a) \(\frac{2}{7}\)   (b) \(\frac{5}{7}\)

Q7. For an event E, \(P(E) + P(\text{not } E) = x\), value of \(x^3 - 3\) is

(a) -2    (b) 2    (c) 1    (d) -1

Q8. The probability that the drawn card from a pack of 52 cards is neither an ace nor a spade is

(a) 9/13   (b) 35/52   (c) 10/13   (d) 19/26

Q9. A dice is rolled twice. The probability that 5 will not come up either time is

(a) 11/36   (b) 1/3   (c) 13/36   (d) 25/36

Q10. One card is drawn at random from a pack of 52 cards. The probability that the card drawn is either red or a queen is

(a) 5/14   (b) 1/14   (c) 5/13   (d) 7/13

Q11. An unbiased die is rolled once. The probability of getting an even prime number is

(a) \(1/3\)   (b) \(2/3\)   (c) \(1/6\)   (d) \(5/6\)
Q12. If a letter is chosen at random from the word “ASSASSINATION”. The probability that the letter is a vowel is

(a) 2/13  (b) 1/13  (c) 6/13  (d) 1/13

Q13. Find the probability that a leap year selected at random will contain 53 Sundays and 53 Mondays.